VOLUME XI SECOND SERIES NUMBER 2

THE PHYSICAL REVIEW

Physics Library
QC
1
.P5812
v.11
no.2

A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS

CONDUCTED BY

THE AMERICAN PHYSICAL SOCIETY

BOARD OF EDITORS

FREDERICK BEDELL, Managing Editor

E. P. LEWIS	N. E. DORSEY	G. K. BURGESS
W. C. SABINE	WM. DUANE	A. D. COLE
A. TROWBRIDGE	O. M. STEWART	A. C. LUNN

FEBRUARY, 1918

The Brightness Sensibility of the Retina. JULIAN BLANCHARD 81

The Moment of Momentum Accompanying Magnetic Moment in Iron and Nickel. JOHN Q. STEWART 100

A Study of the Fluorescence of Certain Uranyl Salts at Room Temperature. FRANCES G. WICK 121

Proceedings of the American Physical Society. 130

Minutes of the Rochester Meeting; The Nature of the Ultimate Magnetic Particle, *Arthur H. Compton and Oswald Rognley*; The Production and Measurement of High Vacua, *J. E. Shrader and R. G. Sherwood*; On the Residual Rays of Rock Salt, *Herbert P. Hollnagel*; The Mathematical Structure of Band Series, II, *Raymond T. Birge*; Images on Silvered Photo-Plate, *C. W. Wagoner*; Emulsions: (A) A New Method for Making Emulsions, (B) Properties of Emulsions, *Wheeler P. Davey*; Note on a Comparison of High-Temperature Scales, *E. P. Hyde and W. E. Forsythe*; Methods of Temperature-Control in Glass-Melting Furnaces, *Clarence N. Fenner*; On Certain Absorption Bands in the Spectra of the Uranyl Salts, *H. L. Howes*; A New Hydrate of Uranium Nitrate; Complete Achromatization of a Two-Piece Lens, *G. W. Moffitt*; A Self-Recording Evaporometer, *Alexander McAdie*; An Instrument for Continuously Recording the Percentage of Saturation and the Weight of the Water Vapor per Unit Volume in the Free Air, *Alexander McAdie*; Rotation of the Pulley in Melde's Experiment, *Arthur Taber Jones*; Comparative Accuracy of Whirled Psychrometer, Assmann Aspiration Psychrometer, Porous Cup Atometers, Hair Hygrographs, Piche Evaporimeter Saturation Deficit Recorder, Open Water Surface Evaporimeter, and Dry and Wet Bulb Thermometers, *Alexander McAdie*; Measurement of Heat Conductivities of Metals at High Temperatures, *Robert W. King*; Bohr's Atom, Zeeman's Effect and the Magnetic Properties of the Elements, *Jakob Kunz*; The Influence of Temperature Upon the Crushing Strength of a Dental Amalgam, *Arthur W. Gray and Paris T. Carlisle, 4th*; A New Formula for the Temperature Variation of the Specific Heat of Hydrogen. *Edwin C. Kemble.*

New Books 159

PUBLISHED BY

THE AMERICAN PHYSICAL SOCIETY

LANCASTER, PA., AND ITHACA, N. Y.

Reprinted by arrangement with the American Physical Society

JOHNSON REPRINT CORPORATION

NEW YORK and LONDON

UNIVERSITY OF MICHIGAN LIBRARIES

THE AMERICAN PHYSICAL SOCIETY

OFFICERS OF THE SOCIETY

President:	H. A. BUMSTEAD	Yale University, New Haven, Conn.
Vice-President:	J. S. AMES	Johns Hopkins University, Baltimore, Md.
Secretary:	D. C. MILLER	Case School of Applied Science, Cleveland, Ohio
Treasurer:	G. B. PEGRAM	Columbia University, New York City
Editor:	F BEDELL	Cornell University, Ithaca, N. Y.
Local Secretary for the Pacific Coast:	E. P. LEWIS	University of California, Berkeley, Cal.

The Council of the Society consists of the president, vice-president, secretary, treasurer, all living Past Presidents and eight elected members as follows:

Past Presidents: A. A. Michelson, A. G. Webster, Carl Barus, E. L. Nichols, Henry Crew, W. F. Magie, Ernest Merritt, R. A. Millikan.

Elected Members: C. E. Mendenhall, G. W Stewart; Irving Langmuir, G. B. Pegram; G. O. Squier, H. A. Wilson; A. L. Day, G F. Hull.

First reprinting, 1961, Johnson Reprint Corporation

Physics - LSA
Johnson
09-17-63
Replace

Second Series. *February, 1918* *Vol. XI., No. 2*

THE

PHYSICAL REVIEW.

THE BRIGHTNESS SENSIBILITY OF THE RETINA.[1]

By Julian Blanchard.

THE eye is able to perceive with ease and comfort a very wide range of light intensities, a range extending over a billion times. It is able to do this because the sensibility of the retina automatically adjusts itself to the stimulus applied, its action being analogous to that of such a physical instrument as a galvanometer with a continuously variable shunt. In this analogy the current through the galvanometer corresponds to the light flux, the scale reading to the brightness sensation produced by the light and the derivative of the scale reading with respect to the current to the sensibility of the retina. The sensation of course cannot be measured directly, but it can be relatively determined by getting a measure of the sensibility at the corresponding stimulus. Since the sensibility is the derivative of the sensation, or scale reading, with respect to the stimulus, the sensation is conversely the integral of the sensibility with respect to the stimulus.

When light falls upon the retina the sensation produced depends upon a number of variables. It is a function of the intensity of the light flux, the length of time it has been acting (before equilibrium is reached), the wave-length, the area and part of the retina affected and the physiological condition of the eye determined by its previous treatment. It would be a difficult matter to determine this general brightness sensation function, but by holding certain factors constant it is easy to obtain a number of limited relations.

The principal object of this investigation is to measure the brightness sensibility of the retina under certain definite conditions. There are three different ways of doing this, or rather three different sorts of sensibility, which are as follows: (1) *Threshold Sensibility*: This is measured by the least brightness that the eye can see. It is proportional to the reciprocal of the least perceptible brightness instantaneously

[1] Communication No. 45 from the Research Laboratory of the Eastman Kodak Company.

substituted for that at which the sensibility is desired and to which the eye has been previously adapted. (2) *Contrast Sensibility*: This is sensibility to brightness difference, or contrast, and is sometimes called photometric sensibility. The reciprocal of the least perceptible difference in brightness between two adjacent fields is taken as proportional to the sensibility at the brightness being used. Of the same nature as this is flicker sensibility, but this will not be considered. (3) *Glare Sensibility*: This is measured by the reciprocal of the brightness that just appears glaring with the eye previously adapted to any given field brightness. It gives an indication of the ability of the retina to stand an overload.

In this paper data will be given on each of these different kinds of sensibility, on the rate of dark adaptation, and on the equilibrium size of the pupil for different field brightnesses.

The literature on visual sensitometry is extremely varied and is too extensive to be reviewed here. Some of the best work that has been done on contrast sensibility is that of König and Brodhun,[1] while Nagel and his pupils[2] have made use of the threshold method in various ways, particularly in measuring the rate of adaptation. In this laboratory Dr. P. G. Nutting has used the threshold method in an improved manner and some preliminary results have been published.[3] In the present work both the contrast and threshold methods are used with the Nutting type of sensitometer.

APPARATUS. METHOD OF MEASUREMENT.

The apparatus used in these experiments is the same as that described by Nutting[4] with some alterations. It may be called a visual sensitometer. It consists of a matte white board B (Fig. 1) about 60 cm. square with an opal glass window T in the center, 3 cm. square, which is illuminated from behind to any desired intensity. For this purpose there is a Nernst filament N focused by a lens L on a slit S, and sliding in metal ways over this slit is an accurately calibrated absorbing wedge W for controlling the intensity. This small square is termed the test spot. Means are provided for moving the wedge by the observer sitting in

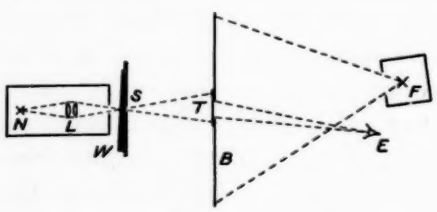

Fig. 1.
The Visual Sensitometer.

[1] A. König, Ges. Abh., pp. 115, 135.
[2] Cf. Helmholtz, Phys. Optik, 3d Ed., Vol. 2, p. 264.
[3] Trans. Ill. Eng. Soc., Vol. 11, p. 1.
[4] Loc. cit.

front of the board and for recording its position. The board is illuminated to any desired intensity by means of a lamp *F* inclosed in a box to the rear of the observer, the illumination being altered in steps by means of neutrally dyed gelatine filters of known transmission placed over the opening in the box. This is called the sensitizing field, or the pre-adaptation field. When using high candle power lamps which glow for a considerable time after the current is cut off, the field was darkened by operating a moving curtain camera shutter in the opening of the box.

The wedge *W* was made by coating a plate of plane glass with a thin layer of neutrally dyed gelatine uniformly increasing in thickness from one end towards the other. Two such wedges, separately calibrated, were placed together with the gelatine faces inside to avoid injury. The density increased from 0.95 one centimeter from the thin end by about 0.4 per centimeter of length, the calibration being carried to a density of 7.5. (Optical density is defined as the logarithm of the opacity, or the logarithm of the reciprocal of the transmission.) The calibration of the wedge for white light is given in Table I. Although very nearly

TABLE I.

Calibration of Absorbing Wedge.

Cm.	1	2	3	4	5	6	7	8	9
Density	0.95	1.36	1.77	2.16	2.59	3.02	3.46	3.92	4.39
Cm.	10	11	12	13	14	15			
Density	4.86	5.34	5.85	6.33	6.85	7.43			

non-selective it was not exactly so and it was therefore calibrated for each of the colors used.

In making an observation on threshold sensibility the procedure is as follows. The eye at *E*, 35 cm. in front of the test spot, is fully adapted to the sensitizing field being used, the field is flashed off and by repeated trials the wedge is set so that the test spot is just visible immediately after extinguishing the field. Or if the threshold is desired at any subsequent time the wedge is moved along so that the test spot is just visible all the while, the position of the wedge being recorded at definite intervals of time as marked off by a sounder. In order to make the determination of the instantaneous threshold easier a white card was held in front of the test spot and at the instant of extinguishing the field this was quickly moved away and back again, giving an exposure of a small fraction of a second in which to decide whether or not the spot was visible. The brightness of the field and of the test spot was measured by means of a portable brightness photometer (modified Beck "lumeter"), recalibrated to read directly in millilamberts over a range from 0.02 to 2 and provided with decimal filters for reading as high as 2,000

ml. The very small intensities, below the range of the instrument, were calculated from the known density of absorbing screen used to cut down a measured higher intensity. In order to obtain the highest field brightnesses the lamp *F* was focused on a small region around the test spot and viewed through a bright tin-lined tube to enlarge the field of brightness. Since this necessitated using only one eye the other measurements were also made with monocular vision.

In all of this work the unit of brightness used is the "lambert," or millilambert, which is 0.001 lambert. It has been officially adopted by the Illuminating Engineering Society and is defined in the 1915 Report of the Committee on Nomenclature and Standards[1] as "the brightness of a perfectly diffusing surface radiating or reflecting one lumen per square centimeter," that is, in accordance with Lambert's cosine law. A perfectly diffusing surface emitting one lumen per square foot will have a brightness of one foot-candle, which is equal to 1.076 millilamberts. A brightness of ten meter-candles is equal to one millilambert. The lambert is to be preferred as a unit of brightness since the foot-candle and the meter-candle are also generally used as units of illumination.

THRESHOLD SENSIBILITY.

In the manner described above the instantaneous threshold was

TABLE II.

Instantaneous Threshold for Different Field Brightnesses.
All values are in millilamberts.

White.		Blue.		Green.		Yellow.		Red.	
Log *B.*	Log *T.*	Log *B.*	Log *T.*	Log *B.*	Log *T.*	Log *B.*	Log *T.*	Log *B.*	Log *T.*
− 6.15	− 5.85	− 7.26	− 6.72	− 6.85	− 6.40	− 5.70	− 5.35	− 4.83	− 4.26
− 5.95	− 5.80	− 6.96	− 6.66	− 6.60	− 6.35	− 5.45	− 5.33	− 4.68	− 4.20
− 5.80	− 5.72	− 6.61	− 6.61	− 6.31	− 6.32	− 5.28	− 5.23	− 4.36	− 4.08
− 5.65	− 5.73	− 6.26	− 6.49	− 6.02	− 6.22	− 4.98	− 5.17	− 4.06	− 4.01
− 5.35	− 5.60	− 5.72	− 6.24	− 5.12	− 5.65	− 4.65	− 5.00	− 3.48	− 3.74
− 5.05	− 5.44	− 4.77	− 5.69	− 4.20	− 5.18	− 3.70	− 4.40	− 2.92	− 3.42
− 4.15	− 4.92	− 3.87	− 5.01	− 3.30	− 4.56	− 2.70	− 3.93	− 2.26	− 3.10
− 3.20	− 4.35	− 2.92	− 4.17	− 2.40	− 3.95	− 2.20	− 3.50	− 1.40	− 2.60
− 2.30	− 3.52	− 2.11	− 3.56	− 1.57	− 3.05	− 1.75	− 3.15	− 0.80	− 2.40
− 1.35	− 2.80	− 1.71	− 3.26	− 1.24	− 2.72	− 1.15	− 2.70	− 0.18	− 2.00
− 0.40	− 2.28	− 1.11	− 2.76	− 0.67	− 2.33	− 0.17	− 2.12	0.37	− 1.70
0.55	− 1.75	− 0.58	− 2.39	0.26	− 1.98	0.10	− 1.90	1.00	− 1.37
1.50	− 1.02	− 0.18	− 2.29	1.03	− 1.64	0.80	− 1.75	1.30	− 1.33
2.00	− 0.75	0.42	− 2.01	1.32	− 1.50	1.10	− 1.52	1.56	− 1.12
2.40	− 0.37	0.66	− 1.86	1.62	− 1.20	1.41	− 1.25	1.81	− 0.97
2.97	0.29	0.97	− 1.61	1.91	− 0.93			2.12	− 0.78
3.30	0.71	1.34	− 1.36						

[1] Trans. Ill. Eng. Soc., Vol. 10, 1915, p. 642.

determined for fields varying in brightness from the highest obtainable (about 2 lamberts for white) down to the threshold itself, with white light and with blue, green, yellow and red. The data are given in Table II. and the curves in Figs. 2 and 6. On account of the great range of values involved it is necessary to plot logarithms of the variables. It is to be noted that − log threshold is proportional to log sensibility, since the reciprocal of the threshold is taken as proportional to sensibility. In Fig. 2 the individual points determining the curves are omitted to avoid

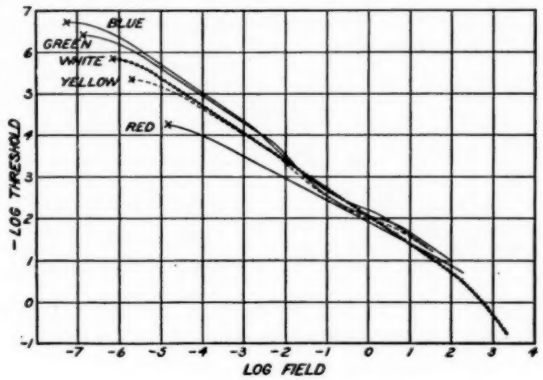

Fig. 2.

Threshold Sensibility for Different Colors.

confusion. The deviations from the smooth curves are no greater than for white light shown in Fig. 6.

The data given are the average of results obtained on three different days, in most cases with several days intervening. In beginning a run it was customary to remain first in darkness about thirty minutes in order to bring the eye into about the same initial condition each time. Observations were made at the threshold first, proceeding to gradually higher intensities, the eye being adapted to each brightness for several minutes before threshold observations were made.

In order to express the results consistently in the same unit of brightness it is necessary to take into account the Purkinje phenomenon. If two fields of different color are illuminated to the same apparent brightness and both cut down by equal amounts the brightness will not decrease in the same ratio. For example, red will grow darker much faster than blue. But at very low intensities it is impossible to measure brightness by any photometric means, and without having a definite measure of the Purkinje effect for the different colors the only feasible way of expressing relative intensities is in fractions of a certain measured intensity above the brightness at which the effect sets in. In these

experiments all the colors were measured photometrically at a brightness of 10 millilamberts, which is safely above the Purkinje effect, and the lower intensities calculated from a knowledge of the filter densities.

In working with the colors both the test spot and the field were colored. These colors were obtained by using filters over the corresponding light sources. The filters were chosen to give a fairly narrow spectral band without having too great a density and their transmission-wave-length curves are given in Fig. 3. In the case of yellow the ordinates have been multiplied by ten. For this color it was necessary to combine two filters, resulting in a rather high density.

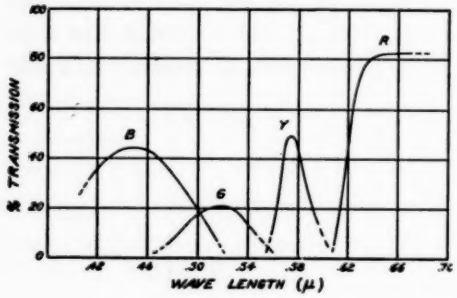

Fig. 3.

Relative Transmission of Color Filters.

The curve for white light is seen to be practically a straight line with the exception of the extreme intensities and may be represented over this portion by the equation,

$$\frac{T}{B} = \left(\frac{B}{B_0}\right)^{-n},$$

in which T is the test spot threshold for any field brightness B, B_0 the absolute field threshold and n a constant. From this, when $B = B_0$ the threshold T is equal to the field itself, although as seen from the curve the test spot threshold is greater than B_0. The reason for this is that the area of the test spot was smaller than the field and the peripheral regions of the retina are more sensitive than the foveal. The angular size of the fovea is between 2 and 3 degrees and the test spot subtended an angle of approximately 5 degrees. If the straight line is extended it will pass through the point where T is equal to the observed B_0.

The lower part of the curve begins to bend at about 100 ml. and in a region between this point and about 2,000 ml., that is, corresponding to bright interiors and outdoor daylight, the curve has a slope equal to unity, which means a constant, and minimum, ratio of T to B over this region. This is analogous to the well-known Fechner constant in contrast sensibility, in that case the least perceptible difference being a nearly constant fraction of the intensity over a much wider range of moderate and high intensities. (At the threshold this fraction is equal to unity and the minimum value is about 0.0175, which is Fechner's constant.) Beyond this region the ratio rises again and at blinding

intensities it would approach unity, that is the instantaneous threshold would be equal to the sensitizing field itself, just as at the other end of the curve.

The equation may be written, taking logs,

$$\log T = (1 - n) \log B + n \log B_0,$$

so that n can be easily determined from the slope of the line. The value of n for white light is here equal to 0.33. The other constants are $B_0 = 0.00000071$ ml. (minimum field threshold), $T_0 = 0.0000014$ ml. (minimum test spot threshold), minimum $T/B = 0.0017$ (about one tenth the Fechner constant).

This curve shows at a glance the very wide range over which the eye can operate and the enormous change in its sensibility. The present experiments cover a range roughly from 10^{-6} to 10^{3} millilamberts, one billion times, and over this range the sensibility as measured by the instantaneous threshold changes more than a million times.

The curves for the different colors are very similar to that for white, the instantaneous thresholds being nearly equal for moderate brightnesses and diverging most for the lowest brightnesses. This is apparently another manifestation of the Purkinje effect, the threshold being least for red and greatest for blue as measured by fractions of the same high intensity. All the color curves except red show a decided dip, indicating a depression of sensibility, in a region roughly between 0.01 and 1 ml., a region corresponding to about the average range of interior brightnesses at night.

For convenience of reference the four principal brightness levels that are encountered are indicated by the crosses in Figs. 5 and 6. These are exteriors at night, 0.001 ml., interiors at night, 0.1 ml., interiors in daylight, 10 ml., and exteriors in daylight, 1,000 ml. These are of course merely rough averages, each lower level being one per cent. of the next higher.

Contrast Sensibility.

The least difference in brightness between two fields that the eye can perceive depends not only upon the brightness of the fields but also upon such factors as their areas and configuration, the previous adaptation of the eye and the time of adaptation to the fields contrasted. With the visual sensitometer contrast sensibility is easily measured with these factors under control. In the experiments as carried out the two fields were obtained by covering the upper half of the test square previously described with a strip of neutral gray gelatine of a certain density, thus affording a fixed contrast between this and the lower half depending upon the transmission of the film. The method of procedure is to adapt

the eye to a certain sensitizing field brightness, flash off the field and then by moving the wedge adjust the test spot brightness until the difference between the two halves is just perceptible after any time desired. In this way time-contrast curves for white light were obtained for several different contrasts, the results appearing in Table III. and

TABLE III.

Time and Brightness to Detect Fixed Contrasts.

Eye initially adapted to 0.1 ml.
Values are log brightness of brighter field, in millilamberts.

Contrast.....	.0.	.39.	.67.	.87.	.97.
0 seconds.....	− 2.80	− 2.63	− 2.40	− 2.10	− 1.20
1 "	− 3.47	− 3.36	− 3.00	− 2.46	− 1.57
2 "	− 3.82	− 3.58	− 3.13	− 2.49	− 1.67
5 "	− 4.30	− 3.74	− 3.22	− 2.48	− 1.69
10 "	− 4.49	− 3.85	− 3.21	− 2.55	− 1.59
20 "	− 4.60	− 3.97	− 3.33	− 2.54	− 1.63
40 "	− 4.89	− 4.06	− 3.46	− 2.67	− 1.73
60 "	− 5.03	− 4.23	− 3.48	− 2.73	− 1.78

Fig. 4. The maximum contrast was secured by using an opaque strip, in which case the actual threshold was determined, and the minimum by means of a thin film of clear celluloid having a transmission of approximately 0.97. The size of the fields was 3 cm. × 1.5 cm. viewed at a distance of 35 cm. (5 degrees × 2.5 degrees), and the brightness of sensitizing field was 0.1 ml. One eye only was used, with natural pupil. The curves are plotted with the brighter of the two contrasted fields as ordinates and show at what brightness a given contrast can just be perceived at any time up to one minute after extinguishing the sensitizing field. It is seen that when the contrast is very small the minimum brightness for it to be perceived does not change much with the time after the first few seconds, but with large contrasts the time factor is very important.

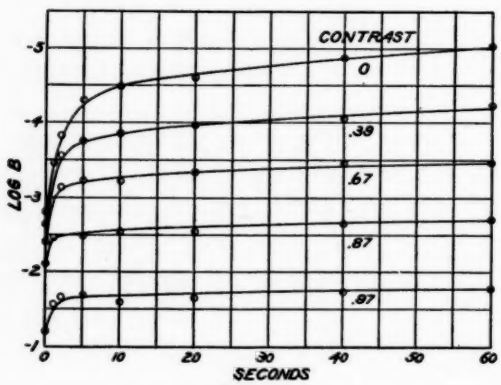

Fig. 4.

Time and Brightness to Detect Fixed Contrasts.

It is easy to obtain from these results the " Fechner fraction," which is the ratio of the least perceptible difference to the brightness at which

it is measured. According to Fechner's law this is a constant over a wide range of moderate and high intensities, but it increases for both extremes. The difference has been generally expressed as a fraction of the lower intensity but it seems more logical to include the increment in the denominator of the fraction, since otherwise the ratio at the absolute threshold is meaningless. Theoretically at the threshold the least perceptible difference is the threshold itself, thus making the ratio here equal to unity. This ratio can be obtained from the above results by merely taking a cross-section of the curves at any given time. The Fechner fraction, $\triangle B/B$, is equal to one minus the contrast ratio and this is plotted against the corresponding value of log B, where B is the higher brightness. A series of such curves is given in Fig. 5 for times of 0, 2, 10 and 60 seconds after extinguishing the sensitizing field, which shows very clearly the effect of time of adaptation and brightness on sensibility to contrast.

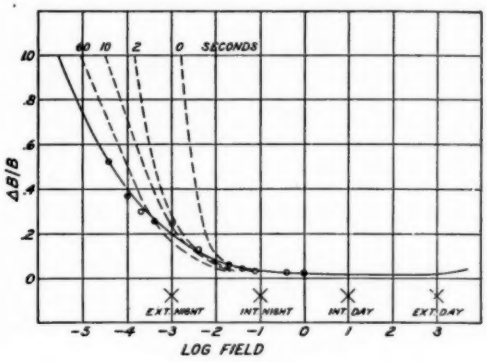

Fig. 5.

Fechner's Fraction.

The Work of König and Brodhun.

König's work (in collaboration with Brodhun) has already been referred to. He determined the least perceptible difference over a very wide range of intensities for white light and for several different wavelengths. His results, however, are deprived of some practical value on account of the uncertainty of his unit of brightness. This he states was the brightness of a magnesium oxide surface illuminated normally by 0.1 sq. cm. of freezing platinum at a distance of one meter and parallel to it. Since this light source is 0.1 the Violle standard, approximately 23 candle power, and since the reflecting power of magnesium oxide is 85 per cent.,[1] this gives as his unit a brightness of 0.20 millilambert. This value, however, is obviously much too high.

For comparison with the results on contrast given above and for obtaining another estimate of König's unit, a partial repetition of his sensibility curve for white light was made on a photometer bench. Two color-matched lamps of about the same candle power were mounted on the bench and by moving the photometer head back and forth the least perceptible difference in brightness between the two halves of the field

[1] Nutting, Jones and Elliott, Trans. Ill. Eng. Soc., Vol. 9, p. 593.

was determined. The low intensities were secured by screening with neutral filters of known density. This is not the same form of apparatus as used by König, although the principle is the same. In his arrangement the contrasted fields were secured by the use of polarized light and crossed nicols. In computing the Fechner fraction König used the ratio of the least difference to the lower intensity and his results have therefore been recalculated using the higher intensity instead, for the reason stated above. His data are given in Table IV., along with the

TABLE IV.

Least Perceptible Difference for Different Field Brightnesses.
All values are in millilamberts.

From König's Data.			Photometer Bench Method.		
Log B.	$\Delta B/B$.	Log ΔB.	Log B.	$\Delta B/B$.	Log ΔB.
3.60	0.0346	2.14	− 0.01	0.021	− 1.69
3.30	.0266	1.72	− 0.41	.025	− 2.01
2.90	.0260	1.31	− 1.11	.032	− 2.60
2.60	.0191	0.88	− 1.41	.042	− 2.79
2.30	.0170	0 53	− 1.71	.060	− 2.93
1.90	.0172	0.14	− 2.41	.131	− 3.29
1.60	.0173	− 0.16	− 3.05	.246	− 3.66
1.30	.0176	− 0.45	− 3.41	.254	− 4.01
0.90	.0178	− 0.85	− 3.71	.302	− 4.23
0.60	.0175	− 1.16	− 4.02	.372	− 4.45
0.30	.0188	− 1.43	− 4.41	.521	− 4.69
− 0.10	.0217	− 1.76			
− 0.40	.0290	− 1.99			
− 0.70	.0314	− 2.20			
− 1.10	.0380	− 2.52			
− 1.40	.0455	− 2.74			
− 1.70	.0560	− 2.95			
− 2.10	.0860	− 3.17			
− 2.40	.110	− 3.36			
− 2.70	.159	− 3.50			
− 3.10	.220	− 3.76			
− 3.40	.274	− 3.96			
− 3.70	.326	− 4.19			
− 4.10	.410	− 4.49			
− 5.54	(1.00)	(− 5.54)			

results of the writer, in millilamberts. The latter are plotted in Fig. 5 as circles, whereas the full line curve is that of König after his values of B have been multiplied by a factor which will bring his curve into as close coincidence as possible with that of the writer. This factor is 0.0040, so that if the contrast sensibility of each observer is approximately the same König's unit of brightness is about 0.0040 millilambert.

König's results were obtained with the eye screened from all light during the test except that of the fields compared, no mention being made of the previous adaptation or the time involved. His contrasted fields were two rectangles, each with apparent sides of 3 degrees and $4\frac{1}{3}$ degrees at the eye, viewed through the natural pupil (presumably, not stated). In the present experiments the fields had apparent sides of 2.5 degrees and 5 degrees, viewed through the natural pupil with the eye continuously screened from all other light, and in the sensitometer the fields were also this size and viewed through the natural pupil. If all conditions are the same the results should be identical with the different forms of apparatus. In Fig. 5 it appears that the sensitometer curves are nearly coincident with the König and photometer bench curves for all adaptation times at the highest intensities used and approach the latter for all intensities as the adaptation time increases. The adaptation time for König's low intensities was probably an hour or more.

As stated in the beginning the reciprocal of the least perceptible brightness difference may be taken as a measure of retinal sensibility (contrast sensibility) and the logarithm of this quantity is plotted in Fig. 6 against the logarithm of the field brightness (the brighter of the two fields). The circles represent the data of König and the crosses the check results of the writer taken on the photometer bench (Table IV.). It is significant that this gives a nearly linear relation of the same general character and range as the threshold method.

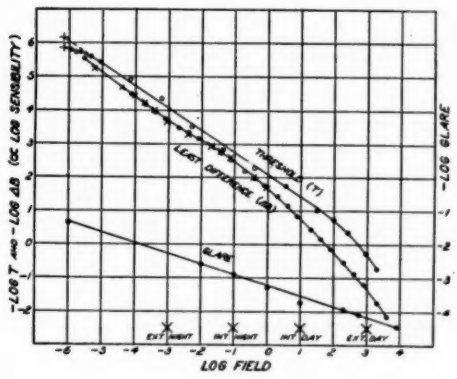

Fig. 6.

Threshold, Contrast and Glare Sensibliity.

Glare Sensibility.

In addition to threshold and contrast sensibility there is a third sort which is not as precisely defined or measured. This is glare sensibility, which is of considerable importance in illuminating engineering. When the eye is adapted to a certain brightness and is then suddenly exposed to a much greater brightness the latter may be called " glaring " if it is uncomfortable and instinctively avoided by the eye. This judgment will naturally depend largely upon the criterion adopted by the observer and different observers may be expected to disagree rather widely. Measure-

ments were made in the following manner with the use of the visual sensitometer. A small mirror was fastened over the test spot so as to reflect into the eye at E (Fig. 1) an image of the opal glass window in the field lighting box F, this constituting the glare source. The angle subtended at the eye by the glare spot was approximately 4.0 degrees. The sensitizing board B was illuminated by means of other lights placed to the rear of the observer. With the eye adapted to a given field brightness the glare lamp F was snapped on and by trial the smallest brightness that was considered glaring was determined. With this apparatus measurements were made with fields from the threshold up to 200 ml. and the highest fields and glare intensities were obtained by using sunlight on white paper and through diffusing window glass with the aid of suitable mirror arrangements. The results of three observers, including the writer, are given in Table V. and the average of all three

TABLE V.

Log Field.	Log Glare.			
	P. R.	P. G. N.	J. B.	Mean.
− 6.0	1.45	0.78	1.81	1.35
− 2.0	2.64	2.65	2.50	2.60
− 1.0	2.74	2.78	3.18	2.90
0.0	3.30	3.30	3.30	3.30
1.0	3.70	3.76	3.62	3.72
2.30	3.87	3.85	4.00	3.91
2.76	4.09	4.11	4.06	4.09
3.91	4.18	4.16	5.02	4.45

plotted in Fig. 6, using the logarithms of the variables. It is seen that the curve is a straight line, the upper limit of which will naturally be where the field is equal to the glare itself. The relation may be represented by the equation

$$\log G = a \log B + \log c,$$

or

$$G = cB^a,$$

where G represents the glare brightness, B the field brightness, and a and c are constants. For the conditions here used $a = 0.32$, $c = 1{,}700$, so that for a given field brightness the brightness of a small area, such as a lamp globe or reflector, which will be considered glaring in the sense here defined may be roughly calculated by taking the cube root of the field brightness and multiplying by 1,700. It is to be expected that with a larger area a smaller brightness would be considered glaring, although no measurements were attempted with different areas.

The curves for the three different kinds of sensibilities are given for comparison in Fig. 6, the negative logarithms of the threshold, least difference and glare being plotted as ordinates since they are proportional to the logarithm of the respective sensibilities. It is observed that although the threshold and contrast methods give very similar results the range of sensibility by the glare method is very much smaller, being only about one thousand times. The latter is essentially different in nature from the other two, being based on a maximal reaction, whereas the former are both based on minimal reactions.

RATE OF ADAPTATION.

The rate at which the eye increases in sensibility on going from light to darkness (dark adaptation) has been studied by Nagel and others.[1] In his experiments the observer entered a dark room from a daylight exterior and noted the time required to just see a given brightness. With the apparatus here employed it is possible to make measurements from the very instant of turning out the field light and it is easier to work under definite conditions. In these experiments the eye was adapted to a given sensitizing field and by adjusting the wedge the threshold was determined at the instant of turning off the field and at intervals of a

TABLE VI.

Rate of Dark Adaptation.

Values are — log threshold, in millilamberts.

Sensitizing Field, Ml.	White				Blue.	Green.	Yellow.	Red.
	0.1.	1.0.	10.	100.	0.1.			
0 secs.	2.79	2.20	1.60	0.90	2.82	2.69	2.61	2.32
1 "	3.82	2.99	2.30	1.66	3.92	4.08	3.84	2.69
2 "	4.13	3.27	2.53	2.00	4.36	4.39	4.17	2.98
5 "	4.50	3.79	3.08	2.46	4.91	4.82	4.41	3.37
10 "	4.75	4.15	3.54	2.64	5.27	5.11	4.65	3.57
20 "	4.96	4.51	3.94	2.88	5.53	5.26	4.78	3.65
40 "	5.16	4.82	4.31	3.20	5.68	5.43	5.02	3.73
60 "	5.32	5.06	4.61	3.84	5.81	5.56	5.09	3.80
2 mins.	5.52	5.22	4.83	4.12	6.00	5.70	5.24	3.92
5 "	5.68	5.52	5.22	4.76	6.23	5.80	5.39	4.02
10 "	5.70	5.68	5.59	5.38				
20 "	5.80	5.81	5.76	5.60				
30 "	5.91	5.86	5.83	5.77				
40 "	6.01	5.97	5.91	5.82				
50 "	5.98	6.02	5.94	5.90				
60 "	6.06	6.04	6.01	5.97				

[1] Op. cit.

few seconds or minutes thereafter. Results for white light with sensitizing fields of 0.1, 1, 10, and 100 ml., up to one hour's dark adaptation, are given in Table VI. and the curves in Fig. 7. Observations were made

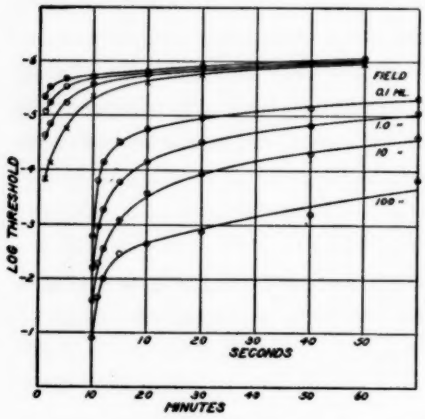

Fig. 7.

Rate of Dark Adaptation, White Light.

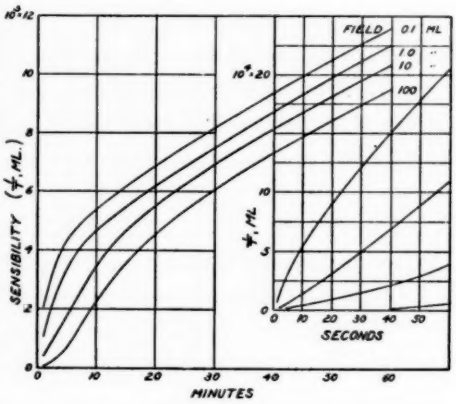

Fig. 8.

Rate of Dark Adaptation, White Light.

in this case with both eyes, natural pupil, test spot 3 cm. square at 35 cm. (visual angle 4.9 degrees). The results are the average of a number of trials made on different days.

In Fig. 7 are plotted the logarithms of the threshold values and these curves show therefore the geometrical increase in the sensibility. The way in which the actual change occurs may be seen better in Fig. 8, where the reciprocal of the threshold, which is proportional to the sensibility, is plotted against the time, these values being taken from the smooth curves in Fig. 7. During the first minute of darkness the sensibility is rather small compared with the total rise and this period is shown on a larger scale in the inserted set of curves. It is seen that when the pre-adaptation field brightness is small the initial rise in sensibility is quite rapid, but with increasing field brightnesses the rise is more and more delayed. The sensibility is still increasing at the end of an hour and continues to rise slightly for several hours. The curves for all brightnesses of course eventually merge into one another.

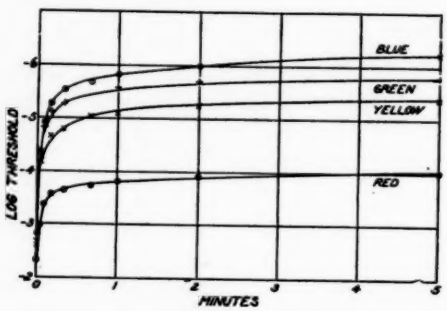

Fig. 9.

Rate of Dark Adaptation, Different Colors.

Similar adaptation curves for a period of five minutes only were taken for the colors blue, green, yellow and red, using the filters previously described. The results are shown in Table VI. and Fig. 9. The sensitizing field was the same color as the test spot and the brightness 0.1 ml. In each case. It is observed that the rise in sensibility is greatest and most rapid for blue and green, which are nearly equal, with red considerably lower and yellow intermediate. The threshold brightnesses in this series were all calculated from the wedge densities necessary to cut down an initial test spot brightness of 0.1 ml., as balanced against white, at which brightness the Purkinje effect is present to some extent.

SIZE OF PUPIL.

The amount of light flux falling upon the retina is directly proportiona to the area of the pupil, which in turn depends upon the brightness to which the eye is exposed. When in bright sunlight the pupil contracts as much as possible to protect the retina from the excessive brightness, and as the brightness diminishes it gradually enlarges and reaches a maximum in complete darkness. The average range is approximately from 2 to 8 millimeters. Several pupillometers have been devised for measuring the diameter but none of them are applicable for all brightnesses, especially for very low intensities. In order to determine the diameter throughout its entire range the method of flashlight photography was used. A large white cardboard was fastened in front of the camera with a hole in the center for the lens. The subject sitting in front of this was adapted to any brightness desired by means of a flood lamp to the rear, such as that used at *F* in Fig. 1. An extra long bellows camera was used so that with the subject close up an enlarged picture of the eye could be obtained, and a chin rest was used to keep the eye in focus. After adapting to the given brightness for at least five minutes the shutter was opened by an assistant and immediately the flash set off and the shutter quickly closed again. A white paper scale stuck on the face in the plane of and close to the pupil enabled the diameter to be accurately measured with the help of a pair of dividers. For the highest brightness used the whole apparatus was moved out of doors. With moderately bright sunlight on white paper a brightness of 2 lamberts was obtained, which is beyond the brightness which the eye can steadily view without discomfort.

When measuring the diameter of the pupil for any brightness it is necessary to take into account whether one or both eyes are open. If for instance the right pupil is being measured this will expand immediately upon closing the left eye and contract when it is opened again. When

the left eye is closed it becomes dark adapted with a consequent expansion of the pupil and the right pupil sympathetically changes in the same direction and thus admits more light. This well-known effect can easily be observed by watching one's eye in a mirror as the other is suddenly opened and closed, the effect being most marked at moderate intensities. A series of measurements under both these conditions is

TABLE VII.

Diameter of Pupil.

Field Brightness, Ml.	Diam., Mm.	
	Both Eyes Open.	One Eye Closed.
0.0	7.4	7.5
0.00015	7.15	7.25
0.01	6.7	7.2
0.6	5.3	6.5
6.3	4.1	5.7
126.	2.6	3.3
355.	2.3	2.9
2,000.	2.0	2.0

given in Table VII. and the curves shown in Fig. 10. The two curves are practically coincident at both extremes and diverge most for brightnesses between 1 and 10 ml., the difference in this region being nearly 1.5 mm.

It is to be noted that the pupil thus measured is not the actual pupil but its image formed by the refracting media in front of it. The iris lies just in front of the lens, and since with different degrees of accommodation the refraction is altered on account of the change of curvature and displacement of the lens, the size of the image of the pupil changes with accommodation. For the eye accommodated to 25 cm. the ratio of the diameters of the image and actual pupil is 1.02, for the unaccommodated eye it is 1.14.[1] In the photographic experiments the white field viewed was about 35 cm. in front of the eye, so

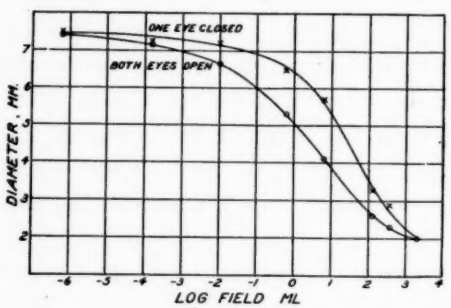

Fig. 10.

Diameter of Pupil for Different Field Brightnesses.

[1] See Helmholtz, Phys. Optik, 3d Ed., and P. G. Nutting, Outlines of Applied Optics, p. 117.

that the size of the pupil image thus measured may be taken as equal to that of the pupil itself, this error being less than that of diameter measurements.

The diameter of the dark adapted pupil varies to some extent with different individuals. Steavenson[1] gives 8.5 mm. as the average of five subjects measured by him by the flashlight method. The limits of the writer are approximately 2 and 7.5 mm. The pupil is also constantly fluctuating over a small range even when the eye is subjected to a fixed brightness. Arrangements are being made to study these variations as well as the rate of opening and closing of the pupil when changing from one brightness level to another by taking a series of pictures with a motion picture camera.[2]

Since the range in diameter of the pupil is roughly from 2 to 8 mm., the ratio of the areas, and consequently the flux upon the retina, is 1 to 16. This means that from the highest endurable brightness to darkness the threshold sensibility increases about 16 times merely on account of the enlargement of the pupil. It has been seen that the total rise in sensibility is more than a million times, so that the increase due to pupil expansion is rather small in comparison with that due to processes going on in the retina itself.

The Flux Density at the Retina.

It is of interest to know the actual flux density at the retina for any given brightness viewed and corresponding size of pupil. This may be approximately calculated as follows.

Consider a small surface of area a_0 sq. mm. normal to the axis of the eye at a distance of u mm. and a brightness of I candles per sq. mm. Treating this surface as a point source of a_0I candlepower, the illumination at the pupil will be a_0I/u^2 lumens per sq. mm., and the flux through the pupil, of area S sq. mm., will be a_0IS/u^2 lumens. Since all of this flux falls on an area a_1 on the retina, the image of a_0 (the small absorption of the eye is here neglected), the flux density at the retina will be a_0IS/a_1u^2 lumens per sq. mm. But from geometrical optics we have $a_0/a_1 = u^2n^2/v^2$, where v is the back focal length of the eye and n is the index of refraction of the medium between the lens and the retina. Hence we have for the flux density, E, in lumens per sq. mm.,

$$E = ISn^2/v^2.$$

[1] Jour. British Astron. Assoc., Vol. 26, p. 303.

[2] Since this was written measurements of the dark adapted pupil for eight subjects have been made, the average being about 8 mm., with values ranging from 7 to 8.7 mm. These results, together with the data on rate of opening and closing, will be published shortly.

If the brightness of a_0 is given in millilamberts, B, instead of candles per sq. mm., then, since by definition 1 ml. $= 1/\pi \times 10^{-5}$ candles per sq. mm., we will have

$$E = 1/\pi \times 10^{-5} BSn^2/v^2.$$

Table VIII. gives a series of values of E for corresponding values of

TABLE VIII.

Field, Ml.	Diameter, Mm.		Effective Area.	Lumens per Sq. Mm.
	From Curve.	Effective.		
B.		*D.*	*S.*	*E.*
0.00001	7.30	8.17	52.2	7.0×10^{-12}
0.001	6.97	7.80	47.8	6.4×10^{-10}
0.01	6.65	7.44	43.4	5.8×10^{-9}
0.1	6.00	6.72	35.4	4.7×10^{-8}
1.0	5.06	5.66	25.1	3.3×10^{-7}
10.	3.86	4.32	14.6	1.9×10^{-6}
100.	2.72	3.04	7.25	9.7×10^{-6}
1,000.	2.08	2.32	4.23	5.6×10^{-5}
2,000.	2.00	2.24	3.94	1.1×10^{-4}

B and S, for the case with both eyes open. In the computations n is taken as 1.34, v is 20.7 mm. (focus for distant vision), and S is the area of the pupil image, in computing which the pupil diameters are taken from the smooth curve in Fig. 10 and multiplied by 1.14/1.02, for the reason previously explained.

For a given brightness of surface the value of E, and hence the apparent brightness to the eye, will change only slightly with the distance of the surface from the eye. For in the above expression for the flux density at the retina, on which the apparent brightness depends, E is directly proportional to S/v^2, or to the solid angle subtended at the image on the retina by the pupil image. On changing the accommodation for near and far objects both S and v^2 change only slightly and in the same direction, so that their ratio remains approximately constant. This independence of the so-called natural brightness upon the distance of the object viewed is borne out by experiment.

SUMMARY.

1. Three different kinds of retinal sensibility have been defined and a new form of apparatus for measuring the sensibility, called the visual sensitometer, has been described.

2. Threshold sensibility (the reciprocal of the least brightness perceptible) has been measured over a wide range of field intensities for white, blue, green, yellow and red light, and a linear relation (with exceptions over certain regions) found between log sensibility and log field brightness.

3. Contrast sensibility has been studied with reference to variations in contrast, brightness and time of adaptation. König's work on the least perceptible difference has been repeated over a limited range and his unit of brightness thereby determined to be approximately 0.0040 millilambert. Recalculating his results, the reciprocal of the least perceptible difference, another measure of retinal sensibility, has been plotted as a function of the field brightness. The curve closely resembles that for threshold sensibility.

4. The least brightness that appears glaring has been determined for all field brightnesses and a linear relation found between log glare and log field.

5. The rate of dark adaptation, with different initial sensitizing brightnesses, has been measured from the beginning of adaptation for white, blue, green, yellow and red light, with the natural pupil.

6. The diameter of the pupil has been measured by means of flash-light photography for different field brightnesses throughout its range, both for monocular and binocular vision, the limits for the writer being approximately 2 mm. and 7.5 mm.

7. From the results of the above measurements the flux density at the retina for a given field brightness and corresponding size of pupil has been calculated for the entire range of vision.

The author wishes to express his thanks and appreciation to Dr. P. G. Nutting, who suggested and largely directed this work, and to Mr. Prentice Reeves, of this laboratory, for their valuable assistance and criticism.

Research Laboratory, Eastman Kodak Co.,
Rochester, N. Y.,
April, 1917.

THE MOMENT OF MOMENTUM ACCOMPANYING MAGNETIC MOMENT IN IRON AND NICKEL.

BY JOHN Q. STEWART.

THE importance of ascertaining whether or not mass is associated with the electric current was recognized by Maxwell, who outlined the principles of three different experimental methods of attacking the problem. Phenomena with which Maxwell was unfamiliar have offered more suitable means of measuring the mass of electricity, and, on account of experimental difficulties, not until quite recently have his methods been successfully applied.

In Electricity and Magnetism, § 577, Maxwell suggests that acceleration of a conductor may generate a current; such currents have been found by R. C. Tolman and T. D. Stewart.[1]

The converse experiment is described in § 574—varying the current might set the conductor in motion. This effect will be discussed later in this article; it probably is too minute to be detected.

The general idea of § 575, namely, that a magnet (or a paramagnetic molecule) acts like a gyroscope, forms the basis of the work of S. J. Barnett, who showed that a rotating cylinder of iron becomes magnetized;[2] and of A. Einstein and W. J. de Haas, who showed that mechanical moment of momentum accompanies magnetic moment.[3]

This was first specifically pointed out by O. W. Richardson, who calculated the relation between the two moments according to the electron theory.[4] If magnetism is due to the motion of charged particles in circular orbits within the atom, a magnetized body must possess internal moment of momentum, the amount of which about any axis is proportional to the component of the magnetic moment along that axis. Working in this laboratory, Richardson made an attempt to verify experimentally his equation,

$$U' = 2\frac{m}{e}M'\frac{1-\frac{e}{E}\frac{M}{m}\frac{A}{a}}{1-\frac{A}{a}}. \qquad (1)$$

[1] Tolman and Stewart, PHYS. REV., VIII., p. 97, 1916.

[2] Barnett, PHYS. REV., VI., p. 239, 1915.

[3] Einstein and de Haas, Deut. Phys. Gesell., 17, p. 152, 1915.

[4] Richardson, PHYS. REV., XXVI., p. 248, 1908.

M' is the magnetic moment, and U' is the corresponding moment of momentum; the multiplying factor (which hereafter we shall call K) is a constant determined by the nature of the rotating sub-atomic corpuscles. These may be positive or negative; for the former M, E, and A, for the latter m, e, and a, respectively, denote the mass, charge in electromagnetic units, and average areal velocity, resolved in the plane perpendicular to the direction of the magnetic intensity. If, as is generally assumed, only the negative electrons are rotating, then

$$K = 2\frac{m}{e} = 1{\cdot}13 \times 10^{-7}. \qquad (2)$$

Barnett's derivation of equation 2 follows: Suppose only negatively charged corpuscles are rotating, one in each orbit; then, if r represents the radius vector, ω the angular velocity, μ the magnetic moment, and u the angular momentum of each system, we have

$$\mu = ea, \qquad a = \tfrac{1}{2}r^2\omega, \qquad \text{and} \qquad u = mr^2\omega = 2ma.$$

Thus

$$\frac{u}{\mu} = 2\frac{m}{e}.$$

Since for any given electron orbit the vectors u and μ are in the same direction, summation through any volume gives

$$\frac{U}{M} = \frac{\Sigma u}{\Sigma \mu} = \frac{u}{\mu} = 2\frac{m}{e}.$$

Richardson thought that the operation of the principle of the conservation of angular momentum would give a means of experimentally detecting the existence of this internal momentum, and of measuring K. If the intensity of magnetization along any axis in a body be changed it follows from (1) that the internal moment of momentum about that axis will correspondingly vary, and from Newton's third law it seems probable that the whole body will tend to rotate in the perpendicular plane. The tendency to rotate will be greater as the moment of inertia of the body about the axis of magnetization is less; this suggests the use of a piece of soft iron wire suspended vertically by a fine quartz fiber within a vertical solenoid. Any rotation may be indicated by the movement of a beam of light reflected from a mirror attached to the wire. When the current through the solenoid is suddenly varied we may expect a temporary vibration of the suspended system.

From time to time, since 1908, unsuccessful efforts to observe this effect with such an apparatus have been made in this laboratory. The most

recent attempt was begun in the spring of 1915 by Mr. Maurice Pate and the writer, under the direction of Prof. H. L. Cooke, and has been carried on by the writer alone. The difficulty was in eliminating the comparatively large disturbances due to the direct action of the field upon the magnetized wire. It was not until after this work was begun that we learned that Einstein and de Haas had succeeded in observing the effect predicted by Richardson, and had determined the value of K to be that which would be due to negative electrons. Barnett, on the other hand, had found a value only half so large. Since the method of Einstein and de Haas was somewhat different from ours (they got rid of disturbing influences by using for the suspension a comparatively tough glass fiber and building up the effect by resonance, with an alternating current), and since, moreover, the numerical data they published seemed inadequate, we thought it worth while to continue the experiment, using our more direct method.

Our results show that Barnett was right in 1915 in estimating the value of K to be only one half that given by (2). A detailed discussion of our work follows.

DESCRIPTION OF THE APPARATUS.

The apparatus included the solenoid, the optical system, various compensating coils, and the wires to be tested.

Fig. 1 is a drawing of the solenoid, with the horizontal scale twice the vertical. The solenoid was built up of three sizes of brass tubing. The two smallest tubes carrying the parallel mirrors, M, M, fitted into one of intermediate diameter, which formed the framework; and over it fitted the larger tube on which was wound the solenoid proper. This consisted of 2126 turns of number 24 double-silk-insulated copper wire, wound in six layers, which varied in diameter from 3.52 to 4.52 cm.; its length was 22.2 cm. The winding was done very carefully: a thin strip of celluloid was wrapped around each layer on its completion, after the wire had been shellacked, in order that the layer next outside might be wound smoothly. The calculated field at the center due to a current of i amperes was 126 i gausses; the calculated self-inductance was 13 henrys; and the calculated resistance was about 25 ohms, agreeing well with the observed value. The temperature rise per minute in the copper wire, neglecting all heat losses, was, in Centigrade degrees, $10i^2$.

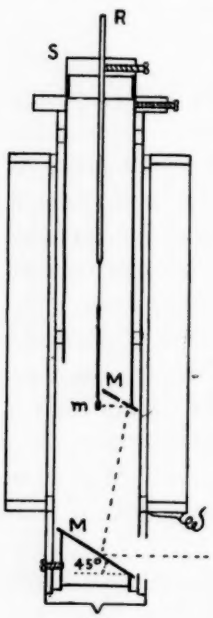

Fig. 1.

To admit of adjustment to the vertical, the solenoid was mounted as is shown in Fig. 2. The lower end rested on a brass strip which could be moved back and forth over the wooden block *B*, which in turn could be rotated into any desired direction. The upper end fitted into a hole in a brass disc fastened into the brass pipe *D*; the hole was half an inch off center. By this arrangement the horizontal component of the solenoid field could be brought into the direction of the block *B*, and then reduced to zero. A screw adjustment was used at the bottom, and one would have been very convenient at the top. In order to keep the whole solenoid from rotating, a projecting rod was held between the prongs of the fork *F*.

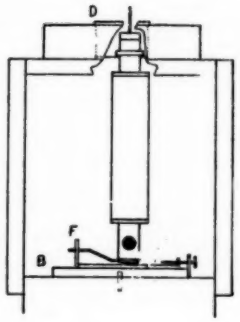

Fig. 2.

The solenoid was mounted in a solidly built box secured to a table with brass screws. One side of the box was left open, and faced north toward another table on which were placed the lamp and scale. Although these two tables were put together with iron bolts this iron was not very near the solenoid, and it caused no trouble.

The optical system was arranged as follows. A Nernst glower was the source of the light, which was reflected by the flat mirrors, *M*, *M* (Fig. 1), up to the middle of the solenoid, to the small mirror, *m*, on the end of the iron wire, and out again to the scale. As the scale distance never was more than sixty centimeters (allowing, of course, for the distance the light travelled down the solenoid), it was unnecessary to make the mirror *m* concave. A number of small flat mirrors, in size about 0.8 mm. by 3 mm., were cut out of thin microscope cover glass, silvered on one face. The band of light reflected on the scale from one of these mirrors was about 2 mm. broad, and sharply enough defined along its vertical edges. Its horizontal edges were not well defined; and in finding the magnetic moment of the wire, when it was necessary to measure vertical displacements of the spot of light, a convex lens had to be placed in front of the Nernst lamp.

The brass tubes to which the mirrors *M*, *M* were attached by soft wax were cut off at exactly 45 degrees; the plane of either mirror could be changed slightly by using extra wax. (Some sort of screw adjustment would have been advantageous.) When things were properly fixed the spot of light had a range of six or seven centimeters on the scale.

Six compensating coils were required to eliminate the earth's magnetic field. A cubical framework was constructed and wound with wire, and fastened to the table with the solenoid at the center. The two

horizontal coils, each of 78 turns, were designed to neutralize the vertical component; they were connected in series to act together. Two other coils of 30 turns each were made to oppose the S. N. component, and two coils of 6 turns each took care of any stray E. W. field. All the coils were approximately square, 60 cm. on a side, and opposite members of a pair were almost that distance apart. The axes of all six met in a point at the center of the cube, where the iron wire was hung. A current of about 0.25 ampere in the proper direction through the vertical and S. N. coils neutralized the earth's field at the center. (78/30 is 2.60, which was the tangent of the angle of dip; in practice, however, each pair of coils was in a separate circuit.)

The field at a point on the axis of a square coil of sides $2a$ at distance y from its plane is

$$H = \frac{8\pi a^2 ni}{(a^2 + y^2)\sqrt{2a^2 + y^2}}, \tag{3}$$

for current i and number of turns n. Since $\partial^2 H/\partial y^2 = 0$ when $y = 0.545a$, a more uniform field at points not very close to the center of the cube would have been secured had the coils in each pair been 32.7 cm. apart.

Two exploring coils were needed for a variety of purposes, as will be explained. The coil C was 90 cm. square, and consisted of 180 turns of number 23 copper wire. It was fastened in a vertical plane on top of a heavy box, so that it could be moved about with the center of the coil at the same height above the floor as the center of the solenoid. A large cardboard scale of degrees was attached, for determining the direction of the normal to the coil with reference to a fixed line on the floor. Since it was of importance to know the number of ampere-turns, the coil was constructed in two divisions, one of 72, the other of 108 turns; the field strengths produced by these were compared, in order to make sure that there were no short circuits. Another exploring coil c was made, similar to C in size and mounting, but of only 10 turns.

The electrical connections were as follows: All the rheostats used were solenoidal, with sliding contacts. The rheostats were kept ten or twelve feet away from the solenoid, except the two employed in eliminating the horizontal component of the earth's field; these were placed within six feet. Each of the earth's field compensating coils was in a separate circuit. All lead wires were closely twisted in pairs. The solenoid was connected in series with a commutator, and could be thrown either into circuit 1 or circuit 2. The E.M.F. in circuit 1 could be varied by a shunt from 0 to 120 volts, and could be either direct or alternating (60 cycles). The E.M.F. in circuit 2 was about 20 volts, and the current was regulated by rheostats. This circuit could be closed through a

switch, or, momentarily, through a mercury contact: a drop of mercury falling through a glass tube made, in passing, instantaneous electrical contact with the amalgamated ends of two copper wires. One or other of the coils C and c could be thrown in circuit with a commutator, rheostats, and a source of direct E.M.F. that could be varied from 0 to 120 volts.

This completes the discussion of the auxiliary parts of the apparatus, but the essential feature, the iron wire itself, remains to be described. The specimen of wire that was being tested was suspended near the center of the solenoid by a quartz fiber from the brass rod R, Fig. 1, which slid in the removable brass stopper S. A number of these brass pieces were constructed in order that several wires could be mounted at one time.

All the wires were pointed at each end, and rolled out straight, in order that the direction of the magnetic moment might lie along the central axis of the wire, and that the mirror, and especially the quartz fiber, might be attached exactly at that central axis. In spite of these precautions there always was a small component of the magnetic moment transverse to the axis of rotation; and this was the cause of the disturbing effects. The quartz fibers were attached to the top, the mirrors to the bottom ends of the wires; the fibers by shellac, burnt hard when an electrically heated, non-magnetic wire was brought near, the mirrors by minute pieces of soft wax. It was found very convenient in mounting the fiber and mirror to have the wire held vertically between the plane parallel ends of two brass rods which slid toward each other through opposite holes in a brass ring; this ring was fastened to a stand immediately below another brass piece that held the stopper S and rod R. The fiber until attached could be handled by a U-shaped piece of wire.

Before mounting the mirror the torsion constant of the fiber was determined; a small brass disc of known moment of inertia was attached centrally to the lower end of the wire, and the period of vibration was observed with the system free from magnetic control. At different times, and to check each other, two such discs were used. Each was about an eighth inch in diameter, and the calculated moments of inertia were 3.61 and 3.53 by 10^{-4}.

The choice of the size of the wire and fiber is of importance, but it can only be made after a consideration of equation 4, which will now be derived.

SIZE OF THE EFFECT.

This is given by equation 4. The equation of motion of the suspended iron wire is

$$I\frac{d^2\theta}{dt^2} + k\frac{d\theta}{dt} + c\theta = 0,$$

where c is the torsion constant of the quartz fiber, k the damping coefficient, I the moment of inertia of the suspended system, and θ the angle through which it is rotated at time t. In practice, $k^2 < 4cI$ and writing $m^2 = 4cI - k^2$, the solution is

$$\theta = \frac{2U}{m} \epsilon^{-\frac{k}{2I}t} \sin \frac{m}{2I} t,$$

for the special case in which alone we are interested, viz., when the rotation is due to an impulse U units of angular momentum which acted when t and θ were zero.

The period of this damped vibration is

$$T = \frac{4\pi I}{\sqrt{4cI - k^2}}.$$

The amplitude of the first swing is

$$\theta' = \frac{U}{\sqrt{cI}} \epsilon^{-\frac{\lambda}{\pi} \tan^{-1} \frac{\pi}{\lambda}},$$

where λ is the logarithmic decrement, usually small. The exponential term can be expanded in a series, and finally we have for δ, the linear deflection at scale distance L corresponding to θ',

$$\delta = \frac{2LKM}{\sqrt{cI}} (1 - 0.500\lambda + 0.227\lambda^2), \tag{4}$$

neglecting $(\lambda/\pi)^3$ and higher powers. KM is substituted for U; M is the change in magnetic moment giving rise, by (1), to the impulse U.

Two other equations we shall need are

$$c = \frac{4\pi^2 I}{T^2} \left(1 + \frac{\lambda^2}{\pi^2}\right), \tag{5}$$

and

$$I = \frac{cT^2}{4\pi^2} \left(1 - \frac{\lambda^2}{\pi^2}\right). \tag{6}$$

To return to the choice of the size of the wire and mirror—it is governed by four considerations.

First. The effect sought for must be large enough for easy observation.

Second. The magnitude of disturbing effects must be small.

Third. The wire must not be so tiny as to require excessive care in manipulation.

Fourth. The wire must not be longer than 8 or 9 cm., or the solenoid field may be non-uniform near the ends; but it should not be very short,

or the demagnetizing factor, as well as the probable value of the transverse moment, will be high.

In (4) the factor $M/\sqrt{cI}$ depends upon the size of the suspended system, and should be as large as practicable. For unit intensity of magnetization this factor becomes $V/\sqrt{cI}$, where V is the volume of the wire. If l is the length and a the diameter, $V/\sqrt{cI}$ is proportional to $la^2/\sqrt{cla^2 \cdot a^2}$ or to $\sqrt{l/c}$, provided the mirror is quite small. The maximum weight the fiber can sustain varies directly as the square, and its torsion constant as the fourth power of its diameter. Calling the latter r, and supposing the fiber loaded to its maximum, or to its maximum divided by a factor of safety, c varies as r^4, and la^2 varies as r^2, or as $\sqrt{c}$; which gives $V/\sqrt{cI}$ proportional to $1/a^2\sqrt{l}$. It is, therefore, of great advantage to use wires of small diameter.

It is disadvantageous to use the smallest possible fibers. In (4) the factor $(1 - 0.500\lambda + 0.227\lambda^2)/\sqrt{c}$ depends upon the size of the fiber, and increases as c decreases; on the other hand, the disturbing direct action of the field, which is not necessarily impulsive, is much less influenced by the increase in damping, and rises rapidly in importance as c is lessened.

Of course, considerably larger values of δ can be obtained if the wire is suspended in a vacuum, but this was deemed unnecessary.

Disturbing Effects and Their Elimination.

The rotation which Richardson predicted does not depend upon the magnetic field produced by the solenoid, but upon the change in orientation of the magnetic molecules which that field causes. The solenoid field, however, as well as the earth's field, acts directly upon the magnetized wire, and the rotation produced by this direct action usually is of a much higher order of magnitude. Such rotation could not be produced if there were no transverse component of the magnetic moment (by symmetry), but that transverse component is never absent. It is possible to find by mathematical treatment the exact value of the rotation θ when the wire is magnetized uniformly at a known angle with its axis of rotation, and is hanging in a uniform field of given strength and direction. The uniform field can exert only a couple on the uniform magnet, and equilibrium is attained when this couple (magnetic) is balanced by the two other couples acting: one due to the twisted quartz fiber (torsional), and the other to the opposite pulls of the tension of the fiber and the weight of the wire (gravitational). Since there are no sidewise forces the fiber remains vertical, and its point of attachment to the wire remains fixed in position. Perhaps the assumption of uni-

form magnetization of the wire is not absolutely correct, but it seems likely that the departures from uniformity are not large enough seriously to invalidate this analysis.

Einstein and de Haas turned from the ballistic method of detecting the Richardson effect to the method of resonance because they believed the elimination of disturbing effects was impossible. It may well have been impossible in the case they had in mind, viz., when the magnetic moment of the wire is reversed by reversing a large, continuous current in the solenoid. Successful elimination of disturbing effects has been attained only when it was the *residual* magnetism of the wire that was varied. It was possible to work with the residual magnetism in this research, since for the wires used the ratio length to diameter was so large that the demagnetizing factors were unimportant, and a high value of M remained after the solenoid field was discontinued if the wire previously had been magnetized to saturation. To reverse such residual magnetization required a large, though only instantaneous, current in the solenoid; but to reduce it to zero a relatively small field (the coercive force) sufficed.

When M is thus varied the behavior of m, the small, accidental transverse moment, requires special comment. Suppose the wire is hanging in the solenoid and the spot of light is reflected on the scale. When the wire is magnetized with the north end up (hereafter, for convenience, simply "magnetized up") and the fiber is untwisted, suppose that m is represented by the vector *01*, Fig. 3, making an angle j with the magnetic meridian. By means of the exploring coil C the position of m at any time can be determined; for when C is placed as indicated in Fig. 3, and only then, will a heavy current through C cause no deflection. When the coil c is placed at right angles to C the magnitude as well as the direction of the vector m can be determined. If now the value of M be reduced to zero and reversed, by a succession of increasing momentary demagnetizing currents through the solenoid, then the vector m will be found to rotate successively to the positions *02*, *03*, *04*, and not until M has been completely reversed will m return to its original direction, *05*. When m is at right angles to its original direction (*03*, *06*), M is zero as nearly as can be determined—its value then is certainly less than 5 per

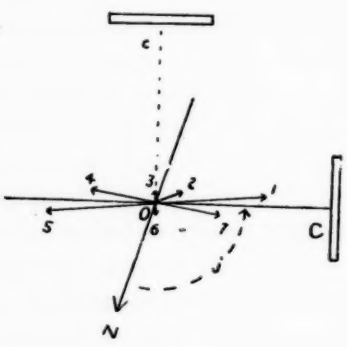

Fig. 3.

cent. of the saturated value. The ratio length *o3* to length *o1* (much exaggerated in the diagram) is of the same magnitude as the ratio diameter of the iron wire to its length.

All the wires tested showed this rotation of *m*, in some cases in the other direction. The cause of the phenomenon is plain enough; *m* is made up of two components, one of which is the horizontal component of the total moment of the wire, while the other, very much smaller, is due to an actual transverse magnetization of the wire, and remains unchanged, except under large fields.

M, then, is certainly zero when *m* has rotated through 90 degrees—that is, when a current through *c* produces no deflection.

The essential condition that must be satisfied before the Richardson effect can be observed is this: the suspended system must be free from magnetic control as regards changes in the value of θ. This result is attained if there is no horizontal field, for it is the horizontal field alone that exerts a couple on the unavoidable transverse component of magnetic moment. The rotation of this component as *M* changes makes necessary, and also possible, an accurate elimination of the horizontal field. Part of the field is that of the solenoid, and can be reversed or reduced to zero at will; the rest of the field is mainly that of the earth.

The currents through the various compensating coils required exactly to neutralize the earth's field are determined by a method of trial and error. The vertical component is eliminated most easily: an alternating current is sent through the solenoid (circuit 1), and gradually reduced to zero; if after this *M* is not zero the current through the compensating coil is changed, and the process repeated. This allows of a very delicate adjustment of the compensating current. The position on the scale occupied by the reflected band of light when the wire is exactly demagnetized is taken as the zero position. The period of vibration is determined; this is the period of the system when free from magnetic control. A current is passed for an instant through the solenoid sufficient to leave the wire magnetized quite strongly; now the spot of light, by adjusting the current in the S. N. and E. W. pairs of compensating coils, is brought back to its zero position. There the fiber is untwisted, therefore what horizontal field (say *h*) is remaining must be in the direction of the transverse component of magnetic moment. By varying together the compensating currents in the horizontally acting coils in such a fashion as to keep the band of light at zero the value of *h* can be changed. When the period of the suspended system becomes that which it had when the wire was demagnetized we can be sure that *h* is zero.

The elimination of any horizontal component of the solenoid field is

carried out in the same manner, still working only with the residual magnetism of the wire. First the block B, Fig. 2, is rotated into the direction of the transverse magnet, OC, Fig. 3. Then the pipe D is revolved until a deflection no longer results on the application of a small solenoid field in the direction of M. Then the wire is demagnetized by the direct current (circuit 2), so that m is at right angles to its previous direction, and the brass plate on the block B is moved until there is no motion of the spot of light on reapplying the demagnetizing field.

The method of taking periods may not completely have got rid of the horizontal earth's field; in that case the deflection when the wire is approximately demagnetized will not be zero. It can be made zero by adjusting the resistances in the compensating circuits; and this is the final adjustment for the horizontal earth's field.

Even if the earth's field is accurately eliminated, and the solenoid field accurately vertical, there remains one disturbing effect: on applying a large solenoid field the wire tends to swing out of its normal position, for ordinarily the direction of its magnetic moment is not quite vertical. Except with a coarse fiber it is impossible to observe the Richardson effect on reversing the residual magnetism of the wire, for this requires too large a field, and the wire is greatly agitated. By far the best method is merely to reduce the residual magnetism to zero; a relatively small field (the coercive force), applied only for an instant, suffices for this.

Aside from the magnetic disturbing effects the only other trouble was caused by shifts in the zero position of the suspended system due to temperature variations. To guard against this a current was never allowed to flow in the solenoid for more than a few seconds at a time; and the zero was redetermined rather frequently. With most of the wires this effect was absent or negligible, with a few it was annoying, it was serious with none.

OBSERVATIONS NECESSARY.

Equation 4 is fundamental, but may be transformed into a more convenient working formula. Let T_1 be the period of the suspended system when a known moment of inertia I_1, large compared with that of the wire, has been added. From (5),

$$c = \frac{4\pi^2 I_1}{T_1^2},$$

for in practice the damping here is negligible. Substituting for c in (6) gives

$$I = I_1 \frac{T^2}{T_1^2}\left(1 - \frac{\lambda^2}{\pi^2}\right),$$

where T is the period of the wire and mirror. Here λ, the logarithmic decrement, cannot be neglected, since I is small. Substituting in (4) these values for c and I,

$$\delta = \frac{K}{\pi I_1}\frac{T_1^2}{T}(1 - 0.500\lambda + 0.278\lambda^2 \cdots)ML. \tag{7}$$

For the two inertia-discs used at different times the values of I_1 were, respectively, 3.61 and 3.53 by 10^{-4}. Substituting the value of K given in (2), and expressing L in meters and δ in millimeters, the magnitude of the constant factor $K/\pi I_1$ comes out 0.100 for inertia-disc 1, and 0.102 for inertia-disc 2. When I was not negligible in comparison with I_1 correction had to be made.

To calculate, then, what δ would be if the value of K were that for the negative electrons, it is necessary to observe T_1, T, λ, L, and M. M is found from an observation of the angle ψ in the vertical plane between the normal position of the wire and its position when a horizontal field H, due to coil C, acts along OC, Fig. 3. If the spot of light reflected on the scale moves vertically a distance p when H is set up, then

$$\psi = \frac{P}{2L}\cos j,$$

supposing the direction of the normal to the mirror is that of the magnetic meridian. (The angle j is measured from the magnetic meridian—see Fig. 3.) Equating the magnetic and gravitational couples,

$$MH = \tfrac{1}{2}Wgl\psi, \tag{8}$$

where Wg is the weight of the wire in dynes, and l is its length. Allowance must be made for the weight of the mirror also; in milligrams this was 0.40 times its area in square millimeters. If $\cos j$ is small M cannot be found, and the mirror must be readjusted.

MANIPULATION.

The wire to be tested was pointed, weighed, measured, and straightened, the inertia disc was attached by a little soft wax, and the fiber was mounted. The wire was placed inside the cubical framework of coils that compensated the earth's field; and the solenoid also was slipped over the wire, which was then demagnetized by gradually reducing to zero an alternating current through the solenoid. The solenoid was removed, and T_1 observed with the suspended system thus freed from magnetic control. The inertia disc was removed, the mirror was attached, and the wire was ready for the test. Usually several suspended systems were constructed at one time.

When the solenoid and compensating coils were back in position, the wire was lowered into the solenoid until the reflection of its mirror could be seen. It was demagnetized by the alternating current, and turned until the spot of light appeared in a central position on the scale. Then the wire was magnetized with the north end up by a momentary current of about an ampere (circuit 1), and the horizontal component of the earth's field was eliminated more or less completely. By the exploring coil C the position of the transverse component m was found, the block B was brought parallel to it, and the lower mirror was turned to bring the spot of light back to its original zero. By turning the pipe D the horizontal component of the solenoid field was brought into the direction of m. The coil c was placed at right angles to the coil C, and the solenoid, with the commutator reversed, was thrown into circuit 2. By trial the instantaneous current just sufficient to demagnetize was found. The criterion for demagnetization was that no deflection be produced by a current in coil c. Once this demagnetizing current—the coercive force—had been determined no further adjustment of the rheostats in circuit 2 was made. Before the coercive force could be found the vertical component of the earth's field had to be eliminated, but this was practically a permanent adjustment. The adjustment of the solenoid to the vertical was next completed, and the final compensation for the horizontal earth's field was effected.

The solenoid was returned to circuit 1, and the wire again strongly magnetized up. The Richardson effect, a sudden throw to the left, could be observed on again demagnetizing by circuit 2.

The horizontal earth's field and the current in the compensating coils would keep varying slightly; and before every observation the deflection had to be reduced to zero by slight changes in the adjustment of the rheostats. With most of the wires, however, the band of light remained nearly steady on the scale, in satisfactory fashion.

After δ had been determined, T, λ, and M were measured. In finding M several readings were made for two or more values of p, and the wire was once or twice remagnetized between times. M always was the same up as down.

THE EXPERIMENTAL RESULTS.

Twenty-four wires were tested—seventeen of iron, six of nickel, and one of silver. The effect sought for was shown by all but the silver wire. Of its reality there can be no question, for it was shown not only by every wire but also by every observation, and the observations agree quantitatively as well as qualitatively.

For nearly all the wires the Richardson effect—a sudden throw to

the left when the wire had been magnetized up, or to the right when the wire had been magnetized down—appeared in company with another impulsive twist, the direction of which was independent of the sense of the magnetization. It can hardly be called a disturbing effect, for it always was of about the same magnitude as the Richardson effect, and obviously the two could easily be distinguished. It varied irregularly in direction and magnitude from wire to wire, and also with the same wire under different conditions; it may have been caused by magnetostriction.

It was impossible to cause variation in what was believed to be the Richardson effect, provided things were not thrown far out of adjustment, when observations could not be taken. Every reasonable test left it unaltered. The same sudden throw was obtained when the demagnetizing field was applied permanently, as when it was allowed to act only instantaneously; but it seemed safer to apply it only instantaneously, in order to eliminate all chance of inaccuracy from imperfect adjustment of the solenoid to the vertical. Changing the time-constant of circuit 2 in the ratio 20/1 left δ unchanged. When one of the wires was rotated through 180 degrees and the light reflected from the other side of the mirror, δ was the same as before.

A very certain disproof of the presence of any effect due to the solenoid field was this: with a rather coarse fiber it was possible to get the Richardson effect when the residual magnetism was reversed, instead of being merely brought to zero; and this admitted of varying the field without changing the flux through the wire. No change in δ was found when the momentary reversing field was increased a hundred per cent. and more. This proved that the effect reached a maximum when the magnet was saturated. That it decreased in ratio with M was also shown. Furthermore one of the wires was so well constructed that it was possible to get the effect on *magnetization*; it was of opposite sign to that obtained on demagnetization, but of exactly the same size.

Table I. shows the results for fifteen of the iron wires and six nickel wires. (Numerical results were not obtained for the first two iron wires tried.) In column 11 δ is the observed deflection reduced to scale-distance 50 cm.; in no case did the scale-distance differ from this by more than a few centimeters. The significance of the results is brought out in column 13, which gives the values of K calculated from the observed values of δ by equation 7. For convenience in interpretation these values of K are expressed as the ratios of the observed K to the value (1.13×10^{-7}) which K would have if only negative electrons were moving, and if all the reaction were effective in imparting angular momen-

tum to the wire. The ratio observed K to calculated K is the same as the ratio observed δ to δ calculated by (7).

TABLE I.

Complete Table of the Experimental Results.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.
No.	Diam. Mm.	Length Mm.	Weight Mg.	Intensity of Magn.	Coerc. Force Gausses.	Obs. I / Calc. I	Obs. I $\times 10^4$.	$10^6 c$.	λ.	δ Mm.	M.	$\frac{K}{1 \cdot 13 \times 10^{-7}}$
Fe:												
1	0.315	52	28.3	950	4.7	1.3	4.7	21.2	0.22	6.6	3.7	0.56
2	.....	52	28.6	870	2.8	1.5	5.3	4.1	38	10.5	3.4	.47
3	.....	52	26.5	590	3.9	1.4	5.1	2.5	67	8.2	2.3	.46
4	.....	48	24.9	470	4.3	1.3	4.3	4.9	43	5.5	1.7	.51
5	.....	35	17.9	700	6.0	1.3	2.7	4.8	37	9.0	1.8	.58
6	.....	34	18.2	880	6.3	1.5	4.2	16.9	19	4.9	2.3	.55
7	0.48	69	84.6	320	3.8	1.1	25.2	4.8	46	5.7	4.0	.52
8	.. ..	75	105.9	340	2.8	1.2	36.6	32.7	19	1.8	4.6	.40
9	0.20	49	10.9	520	5.2	1.5	1.7	10.7	37	3.3	0.78	.58
10	.....	52	12.9	540	5.0	1.7	1.5	5.9	46	7.2	0.87	.84
11	.....	49	11.8	620	5.0	1.8	1.1	7.8	35	8.4	0.93	.88
12	0.13	40	3.6	730	8.8	2.0	1.2	8.0	12	2.1	0.37	.51
13	.....	39	3.4	640	7.4	2.2	0.9	6.8	28	4.4	0.32	1.04
14	.....	31	2.7	460	6.1	2.0	1.0	3.7	48	1.9	0.18	.68
15	.....	52	4.2	670	6.4	2.2	0.9	2.4	51	4.3	0.45	.49
Ni:												
16	0.50	61	100.6	120	30	1.1	35.5	14.0	18	1.2	1.4	.60
17	.....	49	77.6	100	30	1.1	27.2	11.2	26	0.8	1.0	.40
18	0.31	44	28.0	120	28	2.0	7.5	5.2	40	2.5	0.39	1.30
19	.....	52	33.1	80	28	1.8	6.7	5.7	41	2.1	0.33	1.30
20	.....	53	32.6	160	28	1.5	6.0	8.5	32	1.0	0.6	.40
21	0.25	25	10.8	110	38	1.5	2.4	6.2	32	0.5	0.13	.50
Ag:												
22	0.33	39	32.4	.....	.....	1.5	6.5	2.5	48	No effect		

In the remainder of this paper K will be expressed as this ratio; absolute values of K will not be employed.

In Table I. the value taken for δ is in each case an average of six or eight observations. Naturally it is the least accurately determined of all the observed quantities. In some instances the figure in the decimal place was almost guessed at; but the estimate was made before M had been determined and K calculated, and was never revised after K had been figured out.

A few examples are given below of the consistency of the individual observations. "Up" means the wire had been magnetized up; a throw of negative sign corresponds to a clockwise rotation, viewed from

above, which was the direction of rotation of the negative electrons when the wire was magnetized up. The Richardson effect was distinguished from the accompanying irreversible effect in this manner: Suppose that the observed throw when the wire had been magnetized up was δ_1, and when the wire had been magnetized down suppose that the observed throw was δ_2. Then δ', the throw due to the irreversible effect, was $\frac{1}{2}(\delta_1 + \delta_2)$, while the magnitude of δ, the throw due to the Richardson effect, was $\frac{1}{2}(\delta_1 - \delta_2)$. The sign of δ, calculated by this formula, always came out negative—which means that the effect always was in the direction predicted for negative electrons. Examples of the observations of δ follow:

Wire 2, $L = 50$ cm. Up, + 5, 7, 5, 4, 7; Down, + 28, 27, 27, 26, 27 mm. (Average agreement.) Wire 9, $L = 49$. Up, − 5.5, − 5, − 5.3, − 4.7, − 4.5, − 5.7, − 5.0; Down, + 1.5, 1, 1.7, 1.5, 1, 1.5, 1.4. (Average agreement.) Wire 12, $L = 51$. $2\delta = 3.8, 4, 2.5, 5.0, 5.5, 4.5, 3.2, 2.5, 4.5, 3.2, 4.4, 4.3, 5.5, 5.8, 5$. (Worst of all the wires.) For nickel—Wire 20, $L = 50$. Up, − 5.2, − 5.0, − 5.2, − 4.8, − 4.8; Down, − 3.3, − 2.8, − 3.3, − 3.3, − 2.8.

No observation was recorded unless the steady deflection was zero before and after the wire was demagnetized. All the throws were sharp and distinct.

The numbers in column 7 of the table indicate how well each system was constructed. The observed value of I was in every case greater than the calculated value. (Values of I were calculated from the geometrical dimensions of the systems, taking into account the mirrors. For no mirror did the moment of inertia about its own central axis exceed 10^{-6}.) Those systems were best constructed for which the ratios in column 7 are nearest unity. Some of the smaller wires apparently were injured in the process of mounting, and a few of these gave wild values of K. All the wires, however, for which the ratio observed I to calculated I was less than 1.6 gave consistent values of K, and these only should be considered in taking the final averages.

The smallest wires were not intended to improve the mean value of K, but to prove that K does not vary with the diameter and hence that the internal angular momentum actually is proportional to the volume, as (1) demands. This constancy of K seems sufficiently established. The table of results also makes it very evident that the observed K was independent of such factors as the intensity of magnetization, the coercive force, etc.

The value of K seems to be about the same for nickel as for iron. The numerical accuracy of the results is less for nickel, because nickel is far

less magnetizable; and because its coercive force is much larger, which makes it harder to eliminate disturbing effects. The mean value of $K/1.13 \times 10^{-7}$ for the nine iron wires for which the ratios of column 7 are less than 1.6 is 0.51 ± 0.04. For four nickel wires the corresponding mean is 0.47 ± 0.11.

The large departures from the means all are positive. The cause of this phenomenon is unknown (unless it be simply that there is more room for error on that side).

Einstein and de Haas obtained for K a value about twice that found by the writer.[1] They tried only two wires; the first gave $K = 0.75$, and they built a new apparatus. The second gave $K = 0.98$, but they published only seven numerical observations of the value of the "double throw," these all on the same resonance curve—and in taking the mean they discarded the three smallest ones. The ratios observed I to calculated I for their wires were 1.5 and 1.2, respectively. Afterwards another experiment was made by de Haas by a slightly different method; of this later work the writer has seen only the brief account published in Science Abstracts. "An electromagnet is hung from a unifilar suspension with its magnetic axis vertical and performs torsional oscillations. The current is reversed automatically, so that it can be observed whether the magnet has a moment of momentum depending upon and reversed with its magnetism. In one case the moment of momentum was detected and found to be 1.35×10^{-4}. By theory this must be 1.13×10^{-7} the magnetic moment, which gave 1,200 for the magnet instead of 1,400."[2] This would make $K = 0.86$. The resonance method is ingenious, but one cannot be sure that it really does eliminate all disturbing effects. Still another resonance method has been developed by Einstein, but this one is apparently only a lecture-table experiment. The residual magnetism of a suspended iron rod is reversed periodically by an instantaneous current.[3]

MAXWELL'S SECOND EFFECT.

When one of the iron or nickel wires was demagnetized the change in magnetic moment was accompanied by a change of flux and a momentary induced current. It is necessary to show that this current did not produce the sudden throws that were observed.

If the current in the wire moves the wire either this motion is caused by ordinary electromagnetic reaction between the current and the external field, or it is not. Proof has been given in a previous paragraph

[1] Einstein and de Haas, *loc. cit.*

[2] de Haas, Sci. Abs., XIX., p. 351, No. 938, Aug. 25, 1916. K. Akad. Amsterdam, Proc. 18, No. 8, pp. 1281–1299, 1916.

[3] Einstein, Chem. Abs., *11*, p. 1777, 1917.

that the throw δ was independent of the field. (Even the irreversible throw that usually accompanied δ was not caused by the induced current, for it was dependent upon the vertical, not the horizontal, component of the solenoid field.) If the current moves the wire, then, it must move it itself. Such a phenomenon would be of interest, but it does not exist. Since the current is momentary Maxwell's second suggestion does not apply; his second effect can appear only when the current is changed.

According to any electron theory of metallic conduction transference of electricity is by the convection of electrons in the direction opposite to the electric field, and per unit volume there is an exactly equal quantity of positive charge. So long as the current flows steadily a state of statical equilibrium exists, and there is no resultant force of the field upon the body as a whole. When the current is increasing, however, although the positive charge remains immobile, the state of motion of the negative electrons is being subjected to change; and to effect this change a certain amount of the negative field is being used. The result is an unbalanced force in the direction opposite to the negative current, which would give rise to Maxwell's second effect.

Assume the free electron theory. Let there be N free electrons per unit volume; if their average excess velocity over the free path in the direction of the negative current is v, then the current density, i, is Nev. The average momentum per electron is mv, or mi/Ne, and the momentum in volume V is

$$G = \frac{m}{e} Vi.$$

This is the fundamental equation for Maxwell's second effect, on the free electron theory.

Application of this equation to the case of our suspended iron wires shows that the Maxwell effect could not produce an impulse comparable to that caused by the Richardson effect, even if the induced current could be made to keep on flowing (as in a super-conductor), unless demagnetization took place in 10^{-7} second.

Nevertheless, in order to make perfectly certain that it was not some effect of the electrons concerned in conduction that was being observed, a silver wire was tested in the same manner as the magnetic wires. It showed a trace of magnetization, due probably to clinging particles of dust, or to the wax or mirror. The usual adjustments were made, and the steady deflection remained accurately zero when a solenoid field of a hundred gausses was suddenly applied. Shifting of the zero on account of temperature changes was annoying; but δ certainly was less than 0.2 mm., and it seemed to be zero. Of course there was no mag-

netic moment of the silver wire, but, in order to bring the calculation into the same form as for the other wires, it is convenient to assume that the flux, $B = \mu H$, was due to a magnetic moment $M = BV/4\pi$, instead of to the solenoid field H. $V = 3.32 \times 10^{-3}$ cm³., the volume of the wire, and μ is the permeability, which is unity for silver. When $H = 101$, $M = 0.0266$. Accordingly, if K in silver were five times as large as in iron (which is what one would expect from the ratio of the conductivities), the observed value of δ would have been 0.5 mm. Since δ certainly was less than 0.2 mm., we may suppose that the effect is absent in silver.

Einstein and de Haas reported that there is no effect in copper. There probably is no effect in copper, but they did not prove it. Although the conductivity of copper is a few times greater than that of iron the permeability is so much less that it would have required an alternating field of 2,000 gausses, instead of the 50 they used, to get a "double throw" of a millimeter with their apparatus—even if the effect did exist in copper.

SUMMARY OF THE EXPERIMENTAL RESULTS.

A momentum effect such as Richardson predicted for magnetizable substances exists in iron and nickel.

The direction of this momentum is that which would be due to the rotation of negative electrons within the atom; but the magnitude of the effect is only half that which Richardson supposed would result from such rotation of negative electrons.

No such effect exists in silver, whence the effect in iron and nickel cannot be attributed to the conducting electrons.

CONCLUSIONS: THE BEARING UPON THE STRUCTURE OF THE ATOM OF THE VALUE FOUND FOR K.

The internal moment of momentum observed in iron and nickel must be due to the rotation of matter within the atom. It has usually been assumed that only negative electrons are moving, but this assumption leads to an internal momentum twice that observed. It is important to find a reason for the diminished effect.

There are two possible explanations:

1. Negative electrons alone are moving, but cannot react upon the suspended wire with the full effect predicted.

2. Positive as well as negative charges are rotating, in opposite directions.

1. To produce the twist of the suspended wire the rotating electrons must react upon the atom, and the atom, in turn, must react upon the wire as a whole. There are these two chances for loss of part of the

original momentum. Richardson suggested when he predicted the effect that the reaction might take place upon the electromagnetic field, or that the iron atoms might be loose and unable to transmit the momentum to the wire as a whole. The known facts of magnetism, however, render the latter supposition improbable; and if the reaction had taken place upon the electromagnetic system that produced the exciting field the observed effect would not have been independent of the intensity of magnetization.[1]

Barnett's first experiment on the production of magnetization by rotation—an effect the converse of the Richardson effect—agreed with this experiment in giving only half the full effect calculated for negative electrons. The coincidence between the writer's results and those of Barnett not only is evidence of the correctness of both experiments, but also seems to make untenable the loose-atom explanation of the diminished K.

Quite recently[2] Barnett has obtained somewhat larger values of K for steel, nickel, and cobalt. He finds that K has about 80 per cent. of the full predicted value; but the experimental errors are so large that, as Barnett himself states, his results can be considered as agreeing with those of Einstein and de Haas. They certainly agree equally well with those of the writer.

2. We are thus led to the important conclusion that the internal angular momentum in iron and nickel is only half what it would be if negative electrons alone were in motion.

By (1), if expressed as a fraction of $2m/e$,[3]

$$K = \frac{1 - \frac{e}{E}\frac{M}{m}\frac{A}{a}}{1 - \frac{A}{a}}.$$

It follows that

$$\frac{A}{a} = -\frac{1 - K}{K - \frac{e}{m}\frac{M}{E}}. \qquad (9)$$

Experimentally, K has been proved constant with respect to changes in magnetic intensity. Therefore A/a is constant; we proceed to calculate its value.

Assume that the atom is composed of negative electrons and hydrogen nuclei (positive electrons), and that in the iron atom these electrons are

[1] Richardson, The Electron Theory of Matter (1914), p. 396.

[2] Barnett, Proc. Nat. Acad. Sci., 3, p. 178, 1917.

[3] A mistake in sign made by Richardson is corrected here. He wrote $NE = ne$.

not packed so closely together that their mass is appreciably changed. Then, if $E = 1$ and $M = 1$, $e = -1$ and $m = 1/1850$. If the positive and negative electrons, respectively, are moving with angular velocities Ω and ω in circular orbits of radii R and r, it follows that $A = \Sigma\Omega R^2$ and $a = \Sigma\omega r^2$. Substitute these values in (9), and substitute for K the observed value, 0.51. Then

$$\frac{\Sigma\Omega R^2}{\Sigma\omega r^2} = -\frac{1}{3800}. \tag{10}$$

According to Sir Ernest Rutherford's theory of atomic structure, all the positive charges are concentrated in a very small "nucleus" at the center of the atom, while about half the negative electrons are rotating around this nucleus at distances very large compared with its diameter. Equation 10, if true, signifies that the central positive nucleus itself is rotating, but in the opposite direction. A rough calculation based on the assumptions of Rutherford and Bohr, shows that the ratio in (10) will be of the order of magnitude there indicated if the angular velocity of the rotating positive nucleus is about equal (but opposite in sign) to that of the inner ring of electrons.

H. S. Allen[1] has imagined an atom with a rotating positive core surrounded by a ring of revolving electrons, but he assumed $\Sigma\Omega R^2 = \Sigma\omega r^2$. Qualitatively, his assumption of the same sign for Ω as for ω is necessary for his explanation of the magneton.

SUMMARY.

This paper is devoted chiefly to an account of an experiment which showed that iron and nickel, when magnetized, possess internal angular momentum, as was predicted by Richardson in 1908. The magnitude of this momentum can be accounted for if positive, as well as negative, charges are moving within the atom, but in opposite directions. The experimental results of Barnett and of Einstein and de Haas are in qualitative agreement with those described.

The writer is indebted to Professor H. L. Cooke for initial assistance and to Professor K. T. Compton for criticism. It should be possible, with the experimental method described in this paper, to observe the Richardson effect in cobalt and the Heusler alloys, and perhaps also in magnetite.

PALMER PHYSICAL LABORATORY,
PRINCETON UNIVERSITY,
July 14, 1917.

[1] Allen, Phil. Mag., XXIX., p. 714, 1915.

A STUDY OF THE FLUORESCENCE OF CERTAIN URANYL SALTS AT ROOM TEMPERATURE.

BY FRANCES G. WICK.

THE fluorescence spectrum of the uranyl salts consists, as is well known, of a number of bands, more or less well defined, which, at low temperatures, are resolved into bands so narrow as to resemble the lines of a gaseous spectrum. A careful study of the fluorescence spectra of a number of these salts at room temperature made by Nichols and Merritt[1] shows certain common characteristics with regard to location, relative intensity, and shape of the bands. Since data concerning many of the uranyl salts have not been available, a further study of the luminescence of these salts at ordinary temperatures seemed desirable and, at the suggestion of Professor E. L. Nichols, the present work was undertaken. It includes a study of the relative intensities of the bands, determinations of the shape of a few bands which show partial resolution at room temperature and conclusions drawn from measurement of the positions of the crests of the bands of a large number of the salts.

RELATIVE INTENSITIES OF BANDS IN THE FLUORESCENCE SPECTRUM.

The instrument used in this work was a Hilger constant deviation spectrometer similar to the one designed and used by Nichols and Merritt.[1] It is provided with a Lummer-Brodhun cube and two collimators, so that it forms a Lummer-Brodhun spectrophotometer with a constant deviation prism and a drum which reads directly in wavelengths. During the course of this work the calibration of the drum was frequently checked by comparison with mercury and hydrogen lines. For the determination of intensity of fluorescence, the apparatus was set up as follows: The specimen was placed in front of one of the collimator slits of the spectrophotometer and excited to fluorescence by light from a mercury-quartz lamp passed through deep blue glass and brought to a focus by means of condensing lenses. The comparison source was an acetylene flame placed in a carriage in front of the comparison slit. The intensity of illumination of the slit was varied by moving the carriage along a track.

[1] Nichols and Merritt, PHYS. REV., Vol. XXXIII., No. 5, p. 354, Nov., 1911.

A determination of the intensity of the crest of each band of the salt under observation was made by first locating the position of the crest by means of the pointer in the eyepiece of the telescope, then removing the eyepiece and measuring the intensity of fluorescence of this wave-length spectrophotometrically in comparison with the light from the acetylene flame. In order to compare the energy of different parts of the fluorescence spectrum the values obtained for luminous intensity were reduced by means of the energy curve of the acetylene flame determined by Coblentz.[1] The intensity of each crest was multiplied by the ordinate of the energy curve for the corresponding wave-length. The resulting values were reduced to an arbitrary scale in which 10 was taken as the energy of the brightest crest of each salt. Since there was great variation in the brightness of the corresponding crests of different salts the results given in this paper show nothing with regard to the

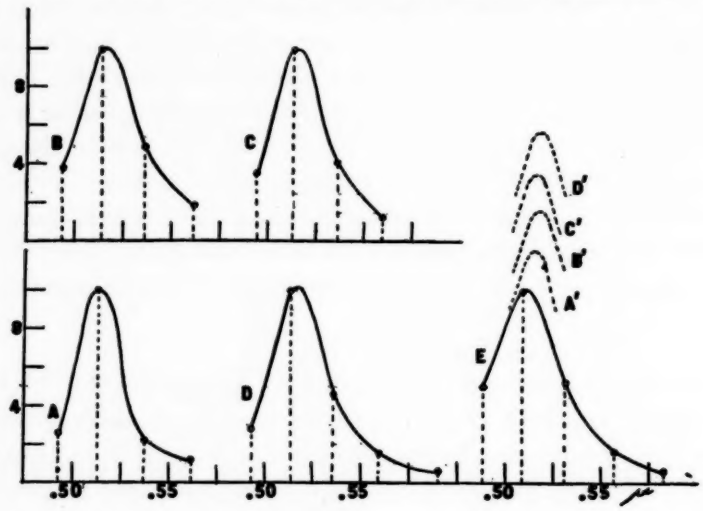

Fig. 1.

Envelopes of fluorescence bands of the double sulphates.

A, uranyl-ammonium sulphate.
B, uranyl-rubidium sulphate.
C, uranyl-potassium sulphate.
D, uranyl-sodium sulphate.
E, uranyl-cæsium sulphate.

A', *B'*, *C'*, *D'*, are the crests of *A*, *B*, *C* and *D* with the base line of each curve two units above that of the curve under it.

absolute intensity of fluorescence in different salts but indicate only the relative intensities of the different bands of the same salt.

The distribution of energy in the bands of the series is shown for a

[1] Coblentz, Bulletin of Bureau of Standards, Vol. 7, No. 2, p. 260.

number of salts in Figs. 1, 3, and 4. The intensities of the crests of the bands expressed in terms of energy are indicated by the dotted lines as ordinates of curves of which the wave-lengths are abscissas. The envelope of these lines shows the distribution of energy among the bands. This energy curve is practically the same shape for all the salts examined and, as has been pointed out by Nichols and Merritt,[1] a curve of this same type represents the distribution of energy in a single fluorescence band of the uranyl salts and also the energy distribution in the broad fluorescence bands of such substances as resorufin and fluorescein. The same form of curve represents the energy in the spectrum of a black body at a temperature of 1259° C. plotted upon a greatly reduced scale.

For the sake of comparison, the envelopes of the different salts were plotted in groups. Fig. 1 shows the envelopes of five of the double sulphates. The maxima of these curves, with the exception of uranyl-cæsium sulphate, Curve *E*, all come at approximately the same position, as may be observed from the superimposed dotted crests *A'*, *B'*, *C'*, *D'*. The maximum of the cæsium salt, curve *E*, is shifted toward the violet from the position of the others. This shift may be accounted for by the fact that the bands of this salt are partially resolved at room temperature and show two crests. The stronger of the two, the one for

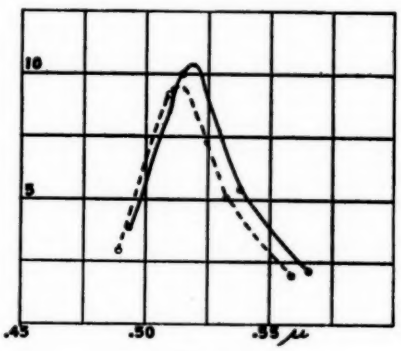

Fig. 2.

Envelopes of fluorescence bands of uranyl sulphate (tri-hydrate).

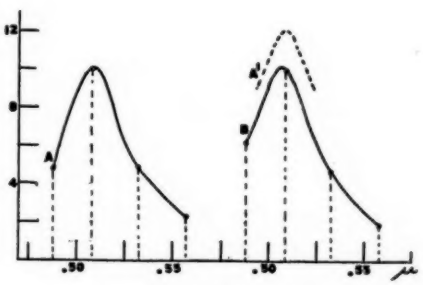

Fig. 3.

Envelopes of fluorescence bands of uranyl acetates.

A, *A'*, uranyl acetate.

B, uranyl-ammonium acetate.

which the intensity was measured, is on the side of the shorter wave-lengths. The envelope of the whole fluorescence spectrum, including both components of the bands, would have its maximum in a position of longer wave-length, which would tend to bring this salt in line with the others.

[1] Nichols and Merritt, PHYS. REV., Vol. XXXIII., No. 5, p. 354, Nov., 1911.

The bands of uranyl sulphate are also partially resolved into two components, the crests of which are well marked, and the observations made upon this salt included the intensities of both crests. A separate envelope was plotted for corresponding components of the bands and these envelopes are shown in Fig. 2. The envelope of the component of shorter wave-length, which, in this case, is the weaker of the two and is indicated by the dotted line, has its maximum in a position of shorter wave-length than that of the other component. A single envelope curve for both sets of components would have its maximum at an intermediate position, which is almost in line with the maxima of Fig. 1.

The results obtained for two of the acetates are shown in Fig. 3. The position of the crest of uranyl-ammonium acetate, Curve *B*, is in a position of shorter wave-length than that of uranyl acetate, Curve *A*.

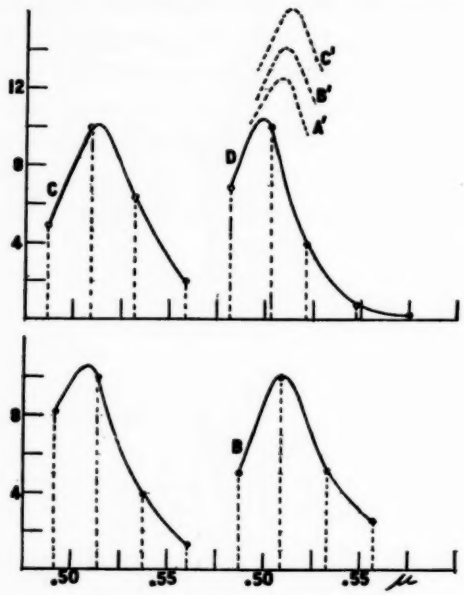

Fig. 4.

Envelopes of fluorescence bands of uranyl nitrates.

A, *A'*, uranyl-potassium nitrate.
B, *B'*, uranyl nitrate (anhydrous).
C, *C'*, uranyl nitrate (tri-hydrate).
D, *D*, uranyl-rubidium nitrate.

Uranyl-ammonium acetate showed a slight degree of resolution but the secondary crest in each band is very weak compared with the one indicated in Curve *B*.

Observations made upon four different nitrates are plotted in Fig. 4. The positions of the crests of the envelopes are approximately the same

with the exception of uranyl-rubidium nitrate, Curve *D*, as is shown by the dotted crests. Here, again, the bands of the salt whose maximum is shifted are partially resolved and settings were made upon the brighter of the two crests which, in this case, also, was the shorter in wave-length.

It appears, in all three of the groups of salts, that the position of maximum is approximately uniform for each group except in the case of salts which are partially resolved. In such cases, the envelope is shifted toward the shorter wave-lengths. The same cause which produces resolution in the bands of the salts appears to shift the energy curves toward the shorter wave-lengths. Upon close inspection of Figs. 1, 3, and 4, it will be observed that the curves of each group which have approximately the same position of maximum show slight differences in position. A study of the curves with reference to molecular weight may be of interest. In Fig. 1, the maximum of the cæsium envelope, Curve *E*, is in the position of shortest wave-length; next in order come the ammonium, rubidium, potassium and sodium salts, Curves *A'*, *B'*, *C'*, and *D'*. In the first three curves, *E*, *A'*, and *B'*, there appears to be a slight displacement of maxima toward longer wave-lengths. In Fig. 3, it will be observed that the uranyl-ammonium acetate, Curve *B*, has its maximum toward the violet of that of the uranyl acetate, Curve *A*. In Fig. 4, the rubidium salt, Curve *D*, has the maximum of shortest wave-length, then in order come uranyl potassium nitrate, uranyl nitrate anhydrous and uranyl nitrate tri-hydrate, Curves *A'*, *B'*, and *C'*. There appears to be some evidence that the increase in the molecular weight causes a shift toward the violet of the energy envelope. This may possibly be due to a tendency of the heavier salts to show partial resolution which might be explained upon the basis of molecular weight.

It has been found by Tutton that for both single and double salts of the alkali metals, several of the optical properties follow the order of molecular weights and that, in the ammonium salts, the NH_4 radical often acts as if it were heavier than the combined weights of its components would indicate, so that its position is close to that of rubidium and sometimes on the side toward cæsium, which appears to be the case here if the shift of the energy curves of a given group of salts is to be connected with the molecular weight.

In order to study the effect of the acid radical upon the energy envelopes, a curve of average position of maximum was taken from each group and the three are plotted for comparison in Fig. 5. It will be observed that there is a decided difference in position of maxima and that the order of position is such that the salts in which the acid radical

is heaviest have their maxima in the position of longest wave-length, the maximum of the sulphates is longest, then the nitrates, then the acetates. The change in the acid radical seems to produce an effect

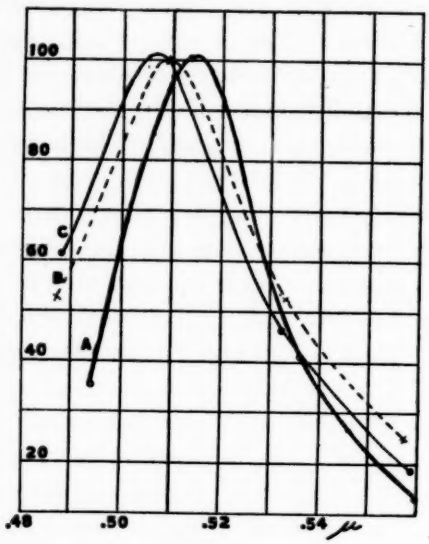

Fig. 5.

Envelopes of fluorescence bands of salts of different groups.
A, uranyl-ammonium sulphate.
B, uranyl nitrate (anhydrous).
C, uranyl-ammonium acetate.

upon the energy curve. An increase in weight causes a shift toward the longer wave-lengths.

SHAPE OF PARTIALLY RESOLVED FLUORESCENCE BANDS.

In most of the uranyl salts at room temperature the bands show a single well-marked crest, but, in some cases, as has been mentioned

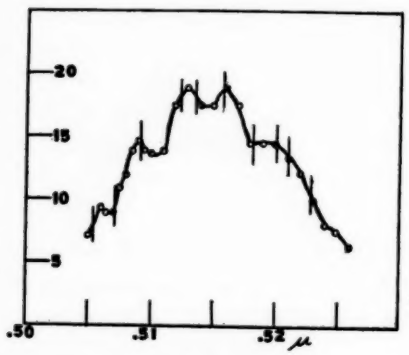

Fig. 6.

Shape of a single fluorescence band of uranyl sulphate (tri-hydrate).
Vertical lines indicate the position of lines in the resolved spectrum at low temperatures.

before, the fluorescence spectrum shows partial resolution and the exact shape of the bands in which resolution has begun is of interest. The form of the bands in several partially resolved salts was determined by making spectro-photometric measurements of the intensity of fluorescence at intervals of 5 or 10 Ångström units. Figs. 6, 7, and 8 show the

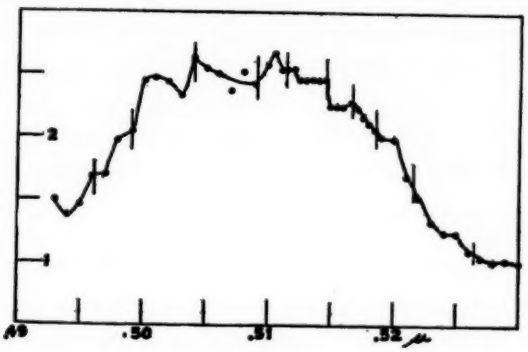

Fig. 7.

Shape of a single fluorescence band of uranyl-potassium nitrate (acid form).
Vertical lines indicate the position of lines in the resolved spectrum at low temperatures.

results of such measurements upon a single band for three different salts. It will be observed that the curves are very irregular in shape, showing many places in which the intensity increases abruptly. The possibility

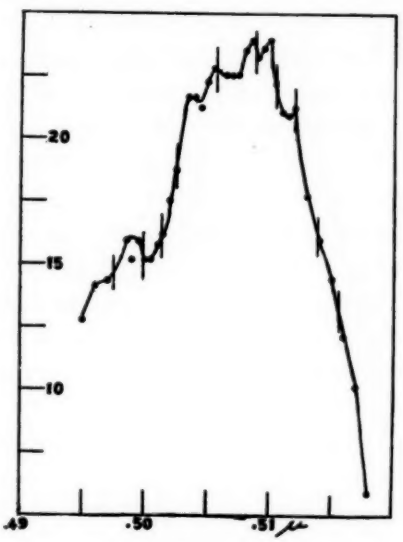

Fig. 8.

Shape of a single fluorescence band of uranyl nitrate (tri-hydrate).
Vertical lines indicate the position of lines in the resolved spectrum at low temperatures.

that these sudden changes in intensity might show the beginnings of the narrow line-like bands which appear in the resolved spectra at liquid air temperatures, was suggested by Professor Nichols. The wavelengths of the lines into which these bands are resolved at low temperatures were obtained from him and the positions of some of these lines are indicated in the figures by short vertical lines. In most cases there is not exact coincidence between the position of the line and that of the maxima of the curves, but this is not to be expected, since there is a shift in the position of the lines of the uranyl salts toward the violet with a lowering of temperature. There appears, however, to be some indication that the lines of the resolved spectrum may have some relation to the irregularities on the curves.

Positions of Fluorescence Bands.

Measurements were made of the positions of the crests of the fluorescence bands in about twenty of the uranyl salts in order to determine whether the characteristics observed by Nichols and Merritt for some of the salts are common to all. The positions of these crests were determined in the usual way by means of the Hilger spectrometer. The crystal, in pulverized form, was strongly illuminated by light from a carbon arc passed through a water cell then through deep blue or purple glass and brought to a focus upon the crystal by means of condensers. The position of the crest was located by means of the pointer in the focal plane of the eye piece. In most of the salts seven bands were observed visually. The sharpness of the crests varies greatly with different salts—in some cases they are narrow and sharp and the results of different sets of observations checked in such a way as to show that the positions could be accurately obtained; in other cases, however, the bands were more than 100 units wide with flat tops, so that no sharp crest was evident and the determinations were not satisfactory. Absorption bands made visible by illuminating the specimen with white or pale blue light were observed and approximately located in some of the salts. Since these bands are broad and poorly defined it was impossible to locate them with any degree of accuracy, either visually or photographically by this method.

The data obtained upon the positions of fluorescence and absorption bands are not published in this paper, but certain conclusions drawn from a study of the measurements made may be mentioned. The results are in agreement with those obtained by Nichols and Merritt for the salts observed by them. The fluorescence bands form a series with a frequency interval which is practically uniform for a given salt. Ab-

sorption bands appear to be a continuation of the fluorescence bands, but with a shorter interval. The two series of bands overlap and certain bands in the violet reversing region appear as absorption or fluorescence bands according to the conditions of illumination.

In the extreme red and violet bands of all the salts observed there is some variation from the uniform interval which may be explained by the fact that these bands are dim and, in the violet region, the absorption overlaps the fluorescence and makes the position of the crest less distinct. The interval is not the same for all the salts, but there appears to be a uniformity in the interval for a given group of salts. For instance, the interval which is characteristic of the nitrates is longer than that of the sulphates.

From the results of the investigation described in this paper it appears that the characteristics of some of the salts observed by Nichols and Merritt, with regard to number of bands, their distribution, and relative intensity are common to all. The spectra of the salts differ in that the intervals between the bands vary and the energy curve is shifted in position for different groups of salts. The degree of resolution at room temperature also varies and it appears that the double salts of highest molecular weight show the highest degree of resolution.

The author wishes to express her sincere thanks to Dr. H. L. Howes for his efficient assistance in making the spectro-photometric measurements described in this paper and to Professor E. L. Nichols whose interest and suggestions have made this work possible. The specimens used belonged to him and a part of the work was done in his laboratory during the summer of 1916.

VASSAR COLLEGE,
POUGHKEEPSIE, NEW YORK.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE NINETIETH MEETING.

THE ninetieth meeting of the American Physical Society was held in Rochester, N. Y., on October 26 and 27. On this occasion the visiting members of the Society were the guests of the Bausch & Lomb Optical Company, the Taylor Instrument Company and the Eastman Kodak Company. The program and various entertainment features were arranged by a committee whose chairman was I. Mayer, of the Taylor Instrument Company, acting in coöperation with Professor F. K. Richtmyer, chairman of the Technical Committee of the Physical Society. The generous hospitality extended included lunch on both days, evening banquet and smoker on Friday evening, automobile transportation to the several companies' works, a theater party for visiting ladies, etc. There were sessions for reading papers forenoon and afternoon of both days. Friday sessions were held at the Hotel Seneca, which was Society headquarters for the meeting. At the conclusion of the afternoon sessions, automobiles conveyed those in attendance to the works of the Bausch & Lomb Optical Company. Demonstrations were there given of methods used in handling large masses of optical glass in the furnaces and of transferring them to the annealing ovens, also of lens grinding and polishing. The Saturday morning session was held at the Taylor Instrument Company's works. Afterwards the visitors were divided into small groups and guides provided to conduct them through the extensive factory. Each visitor was presented with a beautiful souvenir thermometer. Then all were taken by automobile to the Eastman Kodak Company's works at Kodak Park. Here lunch was provided. The afternoon session was held at the Kodak Research Laboratory, and an opportunity given for inspection of the laboratory at the conclusion of the session.

On Friday evening the Society were guests at a dinner at the Hotel Seneca, which included an elaborate musical and patriotic program. At the smoker following Dr. F. E. Wright, of the Geophysical Laboratory, gave an interesting talk on "Optical Glass for Military Purposes," and Major C. E. Mendenhall, of the National Research Council, spoke of the activities of physicists in United States war work. One hundred and fifty-six attended the dinner. The attendance was large at all sessions. A cordial vote of thanks and appre-

ciation for their generous hospitality was extended to the Bausch & Lomb Optical Company, the Taylor Instrument Company, and the Eastman Kodak Company, and to the efficient committee whose excellent plans made the meeting so instructive and interesting.

The program of papers was as follows:

The Production and Measurement of High Vacua. J. E. SHRADER AND R. G. SHERWOOD.

The Nature of the Ultimate Magnetic Particle. ARTHUR H. COMPTON AND OSWALD ROGNLEY.

Bohr's Atom, Zeeman's Effect and the Magnetic Properties of the Elements. JACOB KUNZ. (By title.)

Comparative Accuracy of Whirled Psychrometer, Assman Aspiration Psychrometer, Porous Cup Atmometers, Hair Hygrographs, Piche Evaporimeter, Saturation Deficit Recorder, Open Water Surface Evaporimeter, and Dry and Wet Bulb Thermometers. ALEXANDER MCADIE. (By title.)

Rotation of the Pulley in Melde's Experiment. ARTHUR TABER JONES.

An Instrument for Continuously Recording the Percentage of Saturation and the Weight of the Water Vapor per Unit Volume in the Free Air. ALEXANDER MCADIE. (By title.)

A Self-Recording Evaporimeter. ALEXANDER MCADIE. (By title.)

Complete Achromatization of a Two-Piece Lens. G. W. MOFFITT.

A New Hydrate of Uranium Nitrate; Uranium Nitrate Icositetrahydrate. FRANK E. GERMANN.

A study of the Fluorescence of Certain Uranyl Salts at Room Temperature. FRANCES G. WICK.

On Certain Absorption Bands in the Spectra of the Uranyl Salts. H. L. HOWES.

Optical Range Finders for Military Purposes. HERMANN KELLNER.

Submarine Periscopes. W. B. RAYTON.

An Apparatus for Testing Search Light Mirrors. HENRY KURTZ.

Methods of Temperature-Control in Glass-Melting Furnaces. CLARENCE N. FENNER.

Note on a Comparison of High Temperature Scales. E. P. HYDE AND W. E. FORSYTHE.

A New Formula for the Temperature Variation of the Specific Heat of Hydrogen. EDWIN C. KEMBLE.

The Influence of Temperature Upon the Crushing Strength of a Dental Amalgam. ARTHUR W. GRAY AND PARIS T. CARLISLE, 4th.

High Temperature Measurements. R. C. SCHWARTZ.

Aneroid Barometers. P. R. JAMESON.

Mercury-Steel Capillary Thermometers. J. W. WARD.

Heat Treatment of Mercurial Thermometers. H. Y. NORWOOD.

Emulsions. (*a*) A New Method for Making Emulsions. (*b*) Properties of Emulsions. WHEELER P. DAVEY.

Images on Silver Photo-plate. C. W. WAGGONER.

The Mathematical Structure of Band Series, II. RAYMOND T. BIRGE.

On the Residual Rays of Rock Salt. HERBERT P. HOLLNAGEL.

The Absorption of Near Infra-red Radiation. W. W. SLEATOR.

Measurement of Heat Conductivities of Metals at High Temperatures. ROBERT W. KING.

Standard Turbidity. By title. P. V. WELLS.

Visual Sensitometry. PRENTICE REEVES.

Photographic Sensitometry. L. A. JONES.

Resolving Power. F. E. ROSS AND KENNETH HUSE.

General Outline of Work Being Carried on in Eastman Research Laboratory. C. E. K. MEES.

A. D. COLE,
Secretary.

THE NATURE OF THE ULTIMATE MAGNETIC PARTICLE.[1]

BY ARTHUR H. COMPTON AND OSWALD ROGNLEY.

IT appears highly probable that when a substance is magnetically saturated, all the so-called "molecular magnets" of which it is composed are arranged parallel to the magnetic field. Thus as a substance becomes magnetized the direction of the axes of these elementary magnets ceases to be unordered, and they are turned in a definite direction. If these ultimate magnetic particles are groups of atoms, magnetization must therefore be accompanied by a translation of the atoms, an hypothesis which has been disproved by K. T. Compton and E. A. Trousdale by showing that the Laue diffraction pattern obtained through a magnetic crystal is not affected by magnetization. If these particles are the atoms themselves, the orientation due to magnetization will change the position of the electrons of which the atoms are composed. In virtue of the fact that the intensity of a beam of X-rays reflected from a crystal face depends upon the arrangement of the electrons in the atoms which make up the crystal, such a shift of the electrons should make itself known by changing the intensity of this reflected X-ray beam.

Assume, for example, a crystal composed of atoms of the Bohr type, each atom having all its electrons arranged in the same plane and perpendicular to the magnetic axis. When the crystal is unmagnetized, the electronic orbits will be distributed in all possible planes, so that on the average the electrons will be at an appreciable distance from the mid-planes of their atomic layers. If, however, the crystal is magnetically saturated perpendicular to the reflecting face, the electronic orbits will all lie parallel to this face. The electrons will therefore now be *in* the mid-planes of the layers of atoms which are effective in producing the reflected beam. It can be shown that such a shift of the electrons must produce a very considerable increase in the intensity of the

[1] Abstract of a paper presented at the Rochester meeting of the American Physical Society, October 26 and 27, 1917.

reflected beam of X-rays. On the other hand, if the crystal is magnetized parallel to the reflecting face, the turning of the orbits will carry the electrons farther on the average from the middle of their atomic layers, and a decrease in the intensity of reflection should result.

We have searched in vain for such an effect on the intensity of the reflected beam of X-rays when the reflecting crystal is magnetized. In our experiment a null method was employed. The ionization due to the beam of X-rays reflected from a crystal of magnetite was balanced against that due to a beam of the same wave-length reflected from a crystal of rock-salt, so that a very small change in the relative intensity of either beam could be detected, while variations in the X-ray tube itself had little effect. By means of an electromagnet with a laminated core the magnetite crystal was magnetically saturated, and then demagnetized with an alternating current. The effect of magnetization perpendicular to the plane of the crystal face was investigated for the first four orders. On account of mechanical difficulties the test was made only in the third order spectrum when the crystal was magnetized parallel to the reflecting surface. In no case was any change observed in the intensity of the reflected beam when the crystal was magnetized or demagnetized, though the method was sufficiently sensitive to detect a variation in the intensity of less than 1 per cent.

A direct calculation shows that a displacement of the atoms of 1/200 of the distance between the atoms would have produced a noticeable change in the intensity of the reflected X-ray beam. Our negative result therefore confirms the conclusion of Compton and Trousdale that since the atoms are not appreciably displaced the molecular magnets cannot be groups of atoms.

Similar difficulties are encountered on the hypothesis that it is the atoms which are the ultimate magnetic particles. With an atom of the Bohr type, in which all the electrons are arranged in the same plane, a change in the intensity of reflection as great as 500 per cent. should occur in the higher orders when the crystal is magnetized. Hull has shown that the intensity of X-ray reflection is satisfactorily accounted for if the iron atom is composed of electrons arranged at the corners of cubes of different sizes, but even with this more symmetrical form of atom one would expect a change of some 30 per cent. if the atom is turned around by the magnetic field. In fact, on account of the relatively small number (26) of electrons in an iron atom, it is apparently impossible to assign them any definite arrangement, consistent with what is otherwise known about their distribution, which is so isotropic that a rotation of the atom will not produce a change greater than 1 per cent. in the intensity of the reflected X-ray beam. It is possible to conceive of a perfectly isotropic atom if the electrons, instead of having definite positions or orbits, are arranged as an atmosphere about the nucleus in a wholly unordered manner. An atom so constructed, however, would have no resultant magnetic moment.

It seems to us necessary to conclude that it is neither a group of atoms, such as the chemical molecule, nor the atom itself which is the elementary magnet. We must look rather to the atomic nucleus, as suggested by Merritt,

or to the electron, as proposed by Parson, for the ultimate magnetic particle.

The experimental work and part of the theoretical work on this paper was performed at the University of Minnesota.

WESTINGHOUSE LAMP CO. AND
UNIVERSITY OF MINNESOTA.

THE PRODUCTION AND MEASUREMENT OF HIGH VACUA.[1]

BY J. E. SHRADER AND R. G. SHERWOOD.

THE authors, during their work on the diffusion pump, have designed a pump which they think has points worthy of consideration. Its construction is of the upright form so designed that the condensed mercury returns to the boiler without passing through the high vacuum side. This obviates the objectional feature of Langmuir's early pump in which the condensed mercury came into contact with the hot tube from the boiler, thus producing a mercury vapor blast against the intake side of the pump. The upright form is easier for the glass blower to manipulate in blowing, is more convniente for attaching the water jacket and makes unnecessary the insulation of the stem connecting the boiler to the other part of the pump. The pump is quite effective, pressures lower than 1×10^{-8} mm. Hg. having been attained.

In connection with high-vacua work the Knudsen type of absolute manometer has been chosen as best suited for the measurement of low pressures. A gauge possessing important improvements over those already described in the literature has been constructed. These improvements are (1) the method of supporting the platinum heating strip, (2) the kind of suspension, (3) the manner of suspending and controling the movable vane.

Without heat treatment of a glass system, pressures as measured by this gauge lower than 1×10^{-5} mm. Hg. can not be obtained. With continued heat treatment at 500° C. of the entire hard glass system, pressures lower than 1×10^{-6} mm. Hg have been obtained. Many glass vessels have been exhausted to pressures of the order of 5×10^{-8} mm. Hg.

Testing the pump in connection with the gauge brought out the following relations:

1. With low backing pressure, 3×10^{-4} mm. Hg, the pump begins to operate with 65 watts in the heater and the speed of the pump increases with watts input up to 300 watts after which the increase is much less up to 350 watts. From 350 to 600 watts the speed is practically constant, showing a tendency to decrease at the higher wattage.

2. Critical backing pressure is proportional to watts input over the range of backing pressures from 3×10^{-4} to .6 mm. Hg.

3. From comparison with vapor pressure-temperature curves for mercury, critical backing pressure is a linear function of the vapor pressure of mercury.

[1] Abstract of a paper presented at the Rochester meeting of the American Physical Society, October 26 and 27, 1917.

4. At low backing pressures the speed of exhaustion at wattages above the wattage corresponding to critical backing pressure gradually increases with wattage and comes to a limiting value. With higher backing pressures, the operation of the pump requires higher wattages, but the speed of exhaustion increases more rapidly but approaches the same limiting value. At .6 mm. Hg backing pressure the speed of exhaustion assumes its limiting value with a slight increase in wattage above the wattage corresponding to its critical backing pressure.

WESTINGHOUSE RESEARCH LABORATORY,
EAST PITTSBURGH, PA.,
October 9, 1917.

ON THE RESIDUAL RAYS OF ROCK SALT.[1]

BY HERBERT P. HOLLNAGEL.

IN 1909 Heinrich Rubens, of Berlin, and I published a paper on the determination of the wave-lengths of certain residual rays as obtained by an interferometric method. The curves thus obtained resembled in character the visibility graphs which Michelson had obtained in his study of monochromatic line spectra, that is they showed beat phenomena.

At the time it appeared advisable, from the appearance of the curves, not to push beyond the second minimum, *i. e.*, the first beat. Since then, however, a large number of other residual ray determinations have been made in an entirely similar manner; it seems to have become arbitrarily established that an interference curve should include only the first beat, a procedure evidently somewhat unwarranted and conducive to misleading results.

The energy distributions were likewise approximated as obtainable from the Bjerknes method of calculating the resonance curve. From these it appeared that the distributions consisted of an intense band and a weaker satellite of greater or less wave-length. In fact, this character repeated itself in succeeding investigations so often that Rubens reinvestigated in 1913 the earlier measurements. From this work he concluded that all such bands were due to H_2O vapor absorption for various radiations, saddling a broader band which was of the nature of a resonance curve. In view of what has been stated above, not only may the energy distributions in the first paper be incorrect, but in that of 1913 it is quite possible that they do not show all the absorption bands due to H_2O vapor.

In the light of these possibilities I have deemed it interesting to attempt to push the curves a considerable amount further in order to decide whether the assumptions which are implied in the previous investigations are correct. Such curves extending over four or more beats have been obtained for rock

[1] Abstract of a paper presented at the Rochester meeting of the American Physical Society, October 26 and 27, 1917.

salt. They would seem to indicate that the phenomena are not nearly as simple as previously supposed.[1]

PHYSICAL LABORATORY,
MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
CAMBRIDGE, MASS.
September 8, 1917.

THE MATHEMATICAL STRUCTURE OF BAND SERIES, II.[2]

BY RAYMOND T. BIRGE.

IN the first communication on this subject (Washington meeting, April, 1917) there was proposed a new band series formula, which had been found to hold with the greatest possible accuracy for the main (A_1) series of the 3883 CN band. This formula was to the effect that, if the first frequency differences ($\Delta\nu$) of successive lines be plotted against the ordinary variable "m," there is obtained a hyperbola, running through the origin, or very close to it. The actual frequency is then given by $\nu_0 + \Sigma\Delta\nu$ where ν_0 = frequency of the head of the series.

Although the A_1 series, because of its great length and radical deviations from Deslandres' Law, has been considered standard material for testing new formulæ, the C_1 series of the same band was shown by the author to furnish a far more crucial test. For while the A_1 series fades out just beyond the point of maximum $\Delta\nu$, the C_1 series can be followed for 37 out of the 57 lines forming the portion from the point of maximum $\Delta\nu$ to the (hypothetical) tail.

Uhler has identified the C_1 series from $m = 47$ to $m = 164$. His data indicate the presence of a large number of irregularities. Some of these, however, are only apparent, being due to the confusion of relatively weak C_1 lines with the stronger B_1 and A_1 lines. In order to obtain as consistent data as possible, the author has remeasured, from his own plates, the entire C_1 series, and has succeeded, in addition to removing some of the pseudo-irregularities, in identifying the series over the full interval $m = 6$ to $m = 169$ inclusive. The data used in the following calculations are based jointly on the author's and Uhler's values.

The portion from $m = 16$ down to $m = 6$, embracing those lines where the C_1 series, in perfect analogy with the A_1 series, merges into the corresponding doublet (C_2) series, is uncertain, and the values used show a definite divergence from any simple smooth curve. The C_1 series, aside from this portion, follows the hyperbolic formula with an accuracy commensurate in every way with the A_1 series. The validity of the hyperbolic law is therefore established, and this law seems to furnish one striking evidence of the nature of the fundamental field of force of the molecule.

[1] Grateful acknowledgment is hereby made to the trustees of the Elizabeth Thompson Science Fund as well as to the members of the Rumford Committee of the American Academy of Arts and Sciences for the aid obtained under their respective grants.

[2] Abstract of a paper presented at the Rochester meeting of the Physical Society, October 26 and 27, 1917.

This fundamental field, at certain distinct points, is noticeably disturbed, causing the regular "perturbations." These, while in general equal in number and similarly situated in the five singlet series of this band (as Uhler has pointed out), increase in magnitude from the A_1 to the E_1 series, having values as large as 0.1 A in the C_1 series. Besides the approximately 20 lines forming the perturbations, all lines of the C_1 series between the first regular perturbation at $m = 35$ and the seventh at $m = 148$, show an extremely interesting systematic deviation from the hyperbolic law, viz.:

Beginning at the high-frequency side of any perturbation, the lines have a frequency greater than that expected, by one part in 270,000 to one part in 400,000 (*i. e.*, 0.015 A to 0.01 A). This deviation then decreases linearly, at the rate of about .001 A per line until, when the next perturbation has been reached, it has attained a negative value equal to the initial positive value. In terms of the hyperbola this would mean that the observed frequency differences, outside of the perturbations, all lie on a hyperbola identical with that actually used in the computations, but shifted about 0.001 A nearer the m axis. The difference in the area under the two curves ($169 \times .001$ A) is then exactly compensated by the perturbations which furnish, on the average, frequency differences considerably greater than those computed.

By strict analogy the A_1 series should show similar deviations having a maximum value of 0.003 A. This would be practically masked by the ordinary experimental errors, although the data already presented for A_1 *do* show a slight trace of the expected deviations. It is hoped that these systematic deviations from the hyperbolic law may furnish additional material for any theory of molecular structure.

DEPARTMENT OF PHYSICS,
SYRACUSE UNIVERSITY.

IMAGES ON SILVERED PHOTO-PLATE.[1]

BY C. W. WAGGONER.

IN silvering some mirrors on glass negatives, from which the gelatine had been removed, it was discovered that in a number of cases positive images appeared on the glass. The images developed only when the side of the glass from which the film had been removed was silvered and they had the appearances of faint daguerreotypes.

It was first thought that these images were due to imperfect cleaning but the silvered images reappear after treating the glass with conc. HNO_3, conc. H_2SO_4, Farmer's solution with K_3FeCN_6, 10 per cent. HF, aqua regia, con. NaOH, NaF, and potassium dichromate cleaning solution. Some of the plates were heated in a furnace to the softening temperature of glass without destroying the phenomenon. The original gelatine film was removed by dipping the plates in lye and then subjecting them to steam under pressure. In examining

[1] Abstract of a paper presented at the Rochester meeting of the American Physical Society, October 26 and 27, 1917.

a large number of plates after the film had been removed it was found that on a few, perhaps 5 per cent., a very faint image could be seen by reflected light and when the glass was silvered this image became very pronounced. This faint image could not be removed by the chemical cleaning mentioned above.

It appears that this phenomenon is due to the action of gelatine on the surface of the glass. That a film of gelatine will actually tear pieces of glass from the surface upon which it is placed and allowed to harden is well known to glass manufacturers and is the process used in making the so-called "chipped glass" surfaces.

The images may be accounted for by the presence of the large amount of metallic silver in the shadows on the negative which may reduce this tearing action of the gelatine thus leaving a smoother surface for the silver mirror. The above reasoning would account for the fact that the image is always a positive image.

WEST VIRGINIA UNIVERSITY.

EMULSIONS: (*a*) A NEW METHOD FOR MAKING EMULSIONS. (*b*) PROPERTIES OF EMULSIONS.[1]

BY WHEELER P. DAVEY.

(*a*) The use of gasolene and similar organic solvents for japan is attended by a considerable fire and accident risk. It seemed desirable, therefore, to devise some means of applying japan which did not involve the use of inflammable solvents. With the increased demand for hydrocarbon oils for military purposes, came increased incentive to avoid the use of gasolene and similar solvents in industry. As a result, a method has been devised by which the japan base is emulsified in water and is later deposited from the emulsion upon the surface to be japanned. In the course of this work a new method of making emulsions has been found, and some interesting properties of emulsions have been noted.

All the methods reported in the literature to date for making emulsions involve either a violent mechanical agitation or a grinding action, such as is found in the "homogenizer." It has been found possible, however, to emulsify the oils used in making japan bases (linseed oil, wood-oil, fish oil, etc., their compounds and polymers) by merely heating them with an aqueous solution of an alkali in an inclosed space. The alkali used in the present experiments was ammonia. Since the emulsification can only take place at the interface between the oil and the water, the process may be hastened by providing a large surface. This is easiest accomplished by means of a stirrer with baffles. This stirring is, however, not to be confused with mechanical agitation, for it is carried on at very slow speed, 30 to 60 R. P. M. The fineness of the emul-

[1] Abstract of a paper presented at the Rochester meeting of the American Physical Society, October 26 and 27, 1917.

sions made by this method may be judged from the fact that no difficulty has been experienced in making emulsions of japan base in water which could be put through a cream separator without destroying the emulsion.

Emulsions can be made by this method in two ways, either, (1) all the alkali and a little water may be added to the oil-phase (in this work, the japan base) at first, later adding water gradually, or (2) the whole amount of water may be added to the alkali and oil phase in the beginning. At present, the first way is to be preferred as giving an emulsion of greater fineness.

(*b*) All the emulsions made in this work have formed a scum on the surface if left exposed to the air at room temperature. This has been shown to be due to CO_2, for if air from the room is carefully freed from CO_2 it may be bubbled through an emulsion for a whole day without the formation of scum. If the apparatus for taking out the CO_2 is short circuited by a by-pass a visible scum will form in a few minutes. It has been found, however, that if the temperature of the emulsion is sufficiently low, no scum will form in the presence of CO_2 laden air. An emulsion having over 15 square feet of surface exposed to the air did not scum for weeks when kept at 15°–16° C., but scummed over night at 17° C. At temperatures over the critical temperature, it is as though the CO_2 unites with the alkali on the surface layer, thus forming an electrolyte which breaks the emulsion at the surface, causing a scum. This scum acts as a partial protection for the rest of the emulsion, so that the rate of formation of the scum is greatest when the scum is first beginning to form.

The droplets of the discontinuous phase of the emulsion are negatively charged. This offers a method for separating the discontinuous phase from the continuous phase by the introduction of electrodes into the emulsions. The deposition of the discontinuous phase upon the anode is an example of real electroplating, for the amount deposited is strictly proportional to the product of the current and the time and is independent of the voltage employed except in so far as the voltage affects the strength of the current.

No attempt has been made to accurately measure the velocity of transport of the discontinuous phase under an electric field, but rough measurements indicate that it is not less than 10^{-4} cm. per second per volt per cm. This is of the same order as the velocity of other colloids and ions at the same temperature.

Research Laboratory,
General Electric Co.

Note on a Comparison of High-Temperature Scales.[1]

By E. P. Hyde and W. E. Forsythe.

When comparing the results of high-temperature measurements by different authors there is much uncertainty concerning the scale used. High-temperature scales are for the most part based on the temperatures of

[1] Abstract of a paper presented at the Rochester meeting of the American Physical Society, October 26 and 27, 1917.

the melting points of some chosen metals; gold, copper, and palladium being generally used. The temperatures of the standard melting points are for the most part obtained inside a standard black-body furnace. The temperature scale is extended beyond these points by means of the various radiation laws. All of the laboratories participating in this intercomparison, excepting the Physical Laboratory of the University of Wisconsin, base their temperature scale on the melting point of gold (1336° K.) and extrapolate by means of Wien's equation using for C_2 14350 $\mu \times$ deg. At the Physical Laboratory of the University of Wisconsin the scale is based on the melting point of palladium taken as 1822° K. For C_2 they use 14350 $\mu \times$ deg.

There are three different sources of error, (1) in obtaining the temperature of the melting point, (2) in the blackness of the furnace used, and (3) in extrapolation by means of the radiation law. These various sources of error might well lead to very large uncertainties in the final results. An intercomparison of the temperature scales of the Bureau of Standards, the Physical Laboratory of the University of Wisconsin, the Research Laboratory of the General Electric Company, and Nela Research Laboratory was carried out through measurements made on several tungsten filament lamps sent out by this laboratory. In each of the laboratories the temperature was measured by means of a Holborn-Kurlbaum optical pyrometer using red glass as the monochromatic screen. As the different laboratories used a red glass having a slightly different effective wave-length, a small correction was necessary to reduce the temperatures to the same wave-length. This correction is necessary because the temperature measurements were not made on a black body.

Of the lamps used in the intercomparison all, except T–30–C, had flat filaments about 3 cm. long and about 1½ mm. wide. The exact point at which it was desired to have the temperature measured was indicated either by a pointer, a notch on the supporting lead or a small notch in the filament itself. Three of the lamps were gas-filled and the other two were of the vacuum type, the gas-filled lamps being marked C, while the vacuum lamps are marked with a B. T–30–C, a gas-filled lamp, had a 20 mil (0.5 mm.) filament in the shape of a hairpin loop. As the loop was rather sharp, the exact point at which it was desired to have the temperature measured was easily indicated. T–25–C and T–17–C had their flat filaments horizontal to avoid unequal heatings due to the gas currents within the lamps.

The temperatures were measured in this laboratory, both before the lamps were sent out and after they were returned from each of the other laboratories.

The final results, all reduced to the same value for the melting point of palladium, are given in the following table. As the temperatures obtained are black-body brightness temperatures, it is necessary to give the wave-length to which they correspond.

This intercomparison has been made possible through the coöperation of Dr. Stratton, of the Bureau of Standards, Dr. Mendenhall, of the University of Wisconsin, and Dr. Langmuir, of the Research Laboratory of the General Electric Company.

RESULTS OF INTERCOMPARISON OF TEMPERATURE SCALES.

$C_2 = 14350\,\mu \times \text{deg.}$ $\lambda = 0.665\,\mu$. Melting point of Au. = 1336° K. (pd. = 1828° K.).

T–25–C.

Nela Research Laboratory (10-17-16).	Research Laboratory of General Electric Co. (Schenectady).	Nela Research Laboratory (11-28-16).	Bureau of Standards.	Nela Research Laboratory (4-2-17).
1826° K.	1828	1826		
2214	2214	2215		
2518	2518	2516		

T–16–B.

		1429	1431	1427
1618	1613	1617	1619	1614
1816	1811	1811	1813	1812
2128	2116	2122	2122	2121

T–30–C.

1813	1813	1813	1814	1813
	2307	2304	2302	2303
2756	2752	2752	2762	2752

T–17–C.

Nela Research Laboratory (3-27-16).	Physical Laboratory University of Wisconsin.		Nela Research Laboratory (7-14-17).
	C. E. M.	G. R. G.	
1810	1813	1816	1810
2193	2197	2202	2196
2499	2506	2516	2497

T–18–B.

1599	1602	1605	1597
1806	1816	1819	1807
2105	2119	2123	2107

NELA RESEARCH LABORATORY,
NATIONAL LAMP WORKS OF GENERAL ELECTRIC CO.,
NELA PARK, CLEVELAND, O.,
October, 1917.

METHODS OF TEMPERATURE-CONTROL IN GLASS-MELTING FURNACES.[1]

BY CLARENCE N. FENNER.

THE work described in this paper has been carried out by Dr. F. E. Wright and the writer at the glass-making plant of the Bausch & Lomb Optical Co., Rochester, N. Y.

[1] Abstract of a paper presented at the Rochester meeting of the American Physical Society, October 26 and 27, 1917.

A matter of prime importance in the making of optical glass is proper regulation of furnace temperatures, and to this end the first requisite is a means of determining temperatures by some quick and reliable method. The furnaces in the Bausch & Lomb plant have long been equipped with thermoelements of Pt-PtRh introduced through the furnace walls and leading to a direct-reader conveniently placed for observation by the furnace-men, and the regulation of temperatures has been based upon these, but the method has been unsatisfactory. Probably a chief source of trouble lay in the liability of the thermoelements to contamination from furnace gases. It seemed that an optical pyrometer, constructed on the principle used in the Holborn-Kurlbaum or Morse instruments, would be much better for the purpose, and we obtained one of this type from the Leeds & Northrup Co.

Before putting it into regular use it was considered advisable to obtain information on two points: first, as to whether the calibration-chart supplied with the instrument was correct; and second, whether the radiation given by the furnace walls corresponded to black-body radiation; in other words, whether the apparent temperature was the true temperature.

For the first purpose a long porcelain tube of small diameter, closed at one end, was introduced into the furnace at different temperatures and the true temperature of the end was obtained by a Pt-PtRh thermoelement temporarily inserted. Then the optical pyrometer was sighted through the tube on the hot end, and the readings compared. The results showed that within the limits of error (that is, within a very few degrees) the calibration-chart was correct.

The second matter was investigated as follows: A water-cooled iron tube, several feet in length, was constructed in such a manner that thermoelement leads could be carried from outside through a cool inner tube and about two feet beyond this into a Marquardt porcelain tube, at the end of which the thermo-junction lay. This device, when handled with a little care, could be inserted into the furnace at as high a temperature as 1400° C. and the true temperature of any region determined. By using this simultaneously with the optical pyrometer it was found that at the temperatures at which the important operations of glass-making are conducted (1300°–1400° C.) the determinations of temperature by means of the optical pyrometer agreed almost precisely with those of the thermoelement. At lower temperatures the readings of the optical pyrometer were somewhat high because of reflection of the flames by the glazed walls, but this was of minor importance.

The reliability of the optical pyrometer having been satisfactorily established, it has since been used daily for the control of furnace temperatures. It has been found that the furnace-men could be taught without much difficulty how to use it, so that at night also the temperatures are controlled by it.

Since adopting this method certain troubles in glass-making which were formerly encountered have practically disappeared. The difficulties referred to arose from the fact that if the temperature of melting and fining was a little

lower than it was supposed to be the melt did not fine properly and the resultant glass was full of bubbles, or, in certain cases, might turn milky. If the temperature was too high the pot, even if of best quality, was likely to be corroded and to contribute stones to the glass, or might even be eaten through. In order to avoid running into one or the other of these troubles we are restricted to a rather narrow temperature-range, but by the use of the optical pyrometer close control can be exercised and the difficulties avoided.

GEOPHYSICAL LABORATORY,
WASHINGTON, D. C.,
October 10, 1917.

ON CERTAIN ABSORPTION BANDS IN THE SPECTRA OF THE URANYL SALTS.[1]

BY H. L. HOWES.

PROBABLY Mr. G. C. Stokes[2] was the first investigator to notice that the fluorescence and absorption spectra of the uranyl salts are slightly overlapped.

Morton and Bolton[3] also noticed coincidences in the position of several fluorescence and absorption bands of the uranyl salts.

J. Becquerel and Onnes[4] working at low temperature found several coincidences in the positions of the narrowed bands.

Nichols and Merritt[5] found that the "reversing region" was of considerable length; in the case of uranyl potassium sulphate they were able to reverse the brilliant fluorescence band at 5,130 Å. u. whereas previously the reversals had been limited to the region beyond 5,000 Å. u.

In our study of the uranyl double chlorides Prof. E. L. Nichols and the writer found it possible to reverse a complete group of fluorescence bands lying between 5,080 Å. u. and 4,880 Å. u. The desire to extend this "reversing region" towards the red led the writer to undertake the present investigation. A theory of luminescent radiation very recently proposed by Dr. E. H. Kennard also made the investigation of interest.

Since the crystals are of a greenish yellow color they become rapidly transparent as the light admitted is changed from blue to yellow. This necessitates the use of crystals of increasingly thicker layers to bring out the dimmer absorption bands. To a certain extent the crystal acts as a screen to absorb the blue light which would cause fluorescence, nevertheless it was found necessary to interpose orange or yellow screens of different densities to eliminate fluorescence in a region where ordinarily it is at a maximum. At first the colored glasses made by the Corning Glass Company were used as filters;

[1] Abstract of a paper presented at the Rochester meeting of the American Physical Society, October 26 and 27, 1917.

[2] G. C. Stokes, Phil. Trans., 1852, p. 463.

[3] Morton and Bolton, Chem. News, pp. 47, 113, etc. (1873).

[4] J. Becquerel and Onnes, Leiden Communications, No. 110, 1909.

[5] Nichols and Merritt, PHYS. REV., Vol. 33, Nov., 1911, p. 354.

later, solutions of potassium bichromate of varying concentration. It is evident that the screening must be constantly changed when light from the arc is used as a background for bands of increasingly longer wave-length. It was thought that a beam of monochromatic light could be used as a background and thus obviate exciting the crystal to fluorescence, but a preliminary study by Dr. D. T. Wilber and the writer indicated that such a beam of dispersed light could not be made of sufficient intensity to bring out the dimmer bands.

The rôle played by these new bands in producing fluorescence is a minor one, because they are excessively dim. No doubt if special crystals of great size and exceptional clearness were formed the bands would appear stronger, and more bands could be discovered. The present study has added the reversals of two complete groups to the original group mentioned. For some reason the bands can not be traced as far into the red when the crystal is cooled to $-185°$. It is evident that Stokes's law does not hold and it may be that every fluorescence band has an absorption band of the same wave-length.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
August 31, 1917.

COMPLETE ACHROMATIZATION OF A TWO-PIECE LENS.[1]

BY G. W. MOFFITT.

THE performance of many optical instruments depends largely upon the degree of perfection realized in the achromatization of the thin two-glass lenses which make up the optical system of the instrument. This because of the condition that any system of lenses cannot be truly achromatic unless the individual lenses of the system show complete achromatism. The importance of a definite and complete statement of the conditions which must be fulfilled by the glasses and by the radii of the lens faces is apparent.

Usually this subject is dealt with in terms of partial dispersions, dispersive powers, etc. These, while depending upon the properties of the glasses for their values, are not true constants of the glasses themselves. This is not the case with the Hartmann dispersion constants which are constants of the glasses only.

The conditions for complete achromatization of a two-glass thin lens may be concisely and exactly expressed in terms of the Hartmann constants. The equations show what must be the properties of the glasses if complete achromatization is to be possible, and, granting that the glasses fulfil these conditions, what must be the relations existing between the radii of the lens faces. It is hoped that the following discussion may be of value in the selection of glasses for the design of achromats and in the calculation of the radii to be used.

[1] Abstract of a paper presented at the Rochester meeting of the American Physical Society, October 26 and 27, 1917.

The formula for the focal length of two thin lenses in contact may be written:

$$\frac{1}{f} = (n - 1)A + (n' - 1)B, \tag{1}$$

where $A = 1/r_1 - 1/r_2$, $B = 1/r_3 - 1/r_4$, and n and n' are the indices of refraction of the two glasses for the same wave-length. The Hartmann dispersion formula is

$$n = n_0 + \frac{c}{(l - l_0)^a}, \tag{2}$$

in which n_0, c, l_0, and a, are constants for any particular glass. The value 1.2 for a has been found to hold for practically all optical glasses. Combining equations (1) and (2) gives

$$\frac{1}{f} = \left(n_0 - 1 + \frac{c}{(l - l_0)^a}\right)A + \left(n_0' - 1 + \frac{c'}{(l - l_0')^{a'}}\right)B. \tag{3}$$

In order that f be a constant for all values of wave-length the derivative with respect to l of the right-hand member of the equation must be zero. That is,

$$\frac{-Aca}{(l - l_0)^{a+1}} + \frac{-Bc'a'}{(l - l_0')^{a'+1}} = 0, \tag{4}$$

which may be written

$$\frac{(l - l_0')^{a'+1}}{(l - l_0)^{a+1}} = \frac{-Bc'a'}{Aca} = \text{a constant}. \tag{5}$$

This is true when the derivative with respect to l is zero, or when

$$(l - l_0)(a' + 1) - (l - l_0')(a + 1) = 0. \tag{6}$$

This condition can be fulfilled only when

$$l_0 = l_0', \quad \text{and} \quad a = a'. \tag{7}$$

These are the conditions the two glasses must satisfy if they are to be combined to form a completely achromatized lens. We have here a simple, direct statement to take the place of the clumsy one relating the partial dispersions of the two glasses.

In order to determine the conditions relating the radii of curvature substitute (7) in (5). This gives

$$Ac + Bc' = 0 \tag{8}$$

or, in terms of the radii,

$$c\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + c'\left(\frac{1}{r_3} - \frac{1}{r_4}\right) = 0. \tag{9}$$

If the lens is to be cemented $r_2 = r_3$, and the formula becomes

$$\frac{c}{r_1} + \frac{c' - c}{r_2} = \frac{c'}{r_4}. \tag{10}$$

We have, therefore, a simple method of telling at a glance whether it is possible to completely achromatize a lens of two given glasses. If the respective values of the Hartmann dispersion constants, a and l_0, are the same for the two glasses it is possible to produce a lens showing complete achromatism. The statement that the partial dispersions must be proportional throughout the spectrum is now replaced by the simpler statement relating the Hartmann constants. When the conditions for complete achromatism are fulfilled the ratio of the partial dispersions becomes the ratio of the constants c for the two glasses.

An interesting special case is found when the constants c and c' are equal. That is, when the dispersion curve of one of the glasses is of the same form as that of the other, but displaced parallel to the axis of indices on the dispersion-curve diagram. Equation (10) then becomes

$$r_1 = r_4. \tag{11}$$

The cemented lens would be of uniform thickness measured parallel to the principal axis. Its surfaces might be plane parallel.

As a numerical example let it be required to design a cemented lens of focal length = − 100 cm. One face of the converging element is to be plane. The diverging element will, therefore, be meniscus in form. In a limited list of glasses two are found having the following constants:

	n_0.	l_0.	c.
Dense flint.....	1.70583	2.3185×10^{-5}	0.17254×10^{-5}
Ordinary flint.....	1.58882	2.2906×10^{-5}	0.10113×10^{-5}

It is interesting to note that the textbook from which the list of glasses was taken gives an illustrative example in the design of an achromatic lens. The glasses are selected according to custom. The ones thus selected could not possibly form a lens having the excellence of one made from the glasses tabulated above. This is at once apparent from an inspection of the tabulated Hartmann constants. Substituting the above values of c in Equation (10) gives

$$\frac{0.17254}{r_1} = \frac{0.07141}{r_2}, \quad \text{or} \quad r_1 = 2.416\, r_2,$$

since r_4 = infinity.

To find the values of the radii, substitute in Equation (1), using the values of n_0 and n_0', as these are apt to be known more accurately than any other pair of values of the indices. This gives

$$-0.01 r_1 = 0.70583(1 - 2.416) + 0.58882(2.416),$$

from which $r_1 = -42.31$ cm., and $r_2 = -17.51$ cm.

Using the listed values of the indices this gives $f_D = -100.1$ cm., $f_F = -99.88$ cm., and $f_G = -99.79$ cm. The slight progressive change in the

focal length is, no doubt, due to the discrepancy in the values of l_0, which is a little more than one per cent. in this case. Unless the values of l_0 agree very closely it would be better to compute the radii in the usual way, using the new conditions to determine which glasses will combine with the best results. In general two glasses should be used whose values of the Hartmann constant c differ as much as possible, thereby avoiding great differences in the curvatures of the lens faces.

SAINT LOUIS, MO.,
October 11, 1917.

A SELF-RECORDING EVAPOROMETER.[1]

BY ALEXANDER MCADIE.

THERE is at present no satisfactory instrument for recording evaporation in the free air. The Weather Bureau after many years of experimentation now records evaporation at a number of stations, chiefly west of the Mississippi River, by exposing in a metalic pan of 24 inches radius a water surface the level of which is read by means of a hook gage working in a still-well. By means of a micrometer screw head an apparent accuracy of reading to the thousandth of an inch is obtained; but in reality since but one reading is made in 24 hours, generally about 7 A.M., the true variation in level which may be considerable in the interim, is unknown. The method is objectionable in that gain or loss of the water content due to causes other than evaporation are not separated from the evaporation proper. The entire outfit becomes unserviceable when temperatures are near or below freezing; and in fact the observer is required to store the outfit during the winter months. On the other hand in the summer months the level of the water may vary because of sudden showers and also because of mist or heavy dew. Plainly, evaporation data based upon one observation in 24 hours are of doubtful value in any serious purpose to correlate the rate of evaporation with temperature, humidity and wind movement. At Blue Hill we have been using for some months a weighing device carrying a nearly constant load of water. Evaporation is thus continuously recorded and can be studied in connection with other continuous records such as temperature, wind velocity and direction, vapor pressure, rainfall and air pressure. The instrument is sensitive enough to show the increase due to a heavy dew or what might be called *negative evaporation* occurring when the temperature of the water is lower than that of the lower air and the vapor pressure near saturation. The records are sufficiently detailed for the needs of plant physiologists, engineers and climatologists. The time scale is one centimeter per hour. The evaporation is given in millimeters and at a temperature of 10° C. the weight of the evaporated water is

[1] Abstract of a paper presented at the Rochester meeting of the Physical Society, October 26 and 27, 1917.

approximately one gram for every 10 square centimeters of water exposed. The maximum rate thus far recorded is 1 millimeter per hour, which occurred on July 30, 1917, 2 to 3 P.M. and 3 to 4 P.M. At this time the temperature was 309 A. (1130 N. or 96° F.), the percentage of saturation as determined by an Assmann ventilated psychrometer varied from 37 to 48 per cent., the wind 270° (*i. e.*, from the west) and the velocity 14 meters per second.

It is hardly necessary to add that these continuous records show plainly the difference in the rate of evaporation between daylight and night hours and also are of great value in connection with the study of land and sea breezes.

BLUE HILL OBSERVATORY.

AN INSTRUMENT FOR CONTINUOUSLY RECORDING THE PERCENTAGE OF SATURATION AND THE WEIGHT OF THE WATER VAPOR PER UNIT VOLUME IN THE FREE AIR.[1]

BY ALEXANDER MCADIE.

THE instrument consists of two metallic thermometers, one covered with the usual clean linen constantly wet, thus making a dry and wet bulb set. There is also mounted on the axis of rotation of the thermometers a stretched bundle of hygroscopic hairs, slightly separated and very sensitive. This records percentage of saturation for the particular temperature. Relative humidity as ordinarily determined and recorded means nothing unless the temperature of both air and water surface be given. It is a ratio with the important terms suppressed. Yet we find nearly all meteorological bureaus publishing long tables of relative humidity and these data are used by physicians and others in their discussions of climatic influences. It would be better to give the absolute humidity or weight of the vapor. In the present instrument this is given by means of a record sheet on which is printed the saturation weights for the various temperatures. One reads for any minute the percentage of saturation multiplying by the indicated weight for saturation as shown by the record. The instrument is a modification of an earlier one by the writer called a saturation deficit recorder, since it is easy to ascertain the difference between the weight present and the saturation weight.

Such an instrument, it is thought, will be of some service in sick rooms as well as drying and curing rooms, since it gives a twenty-four-hour record of temperature, dew-point, percentage of saturation and weight in grams per cubic meter of the vapor.

BLUE HILL OBSERVATORY OF HARVARD UNIVERSITY.

[1] Abstract of a paper presented at the Rochester meeting of the Physical Society, October 26 and 27, 1917.

Measurement of Heat Conductivities of Metals at High Temperatures.[1]

By Robert W. King.

In the Physical Review for December, 1915, the writer described an attempt to realize experimentally what in theory is a very simple set of conditions for the measurement of the heat conductivities of metals at high temperatures. Recently the work has been definitely abandoned because of other matters, but since the above article was published considerable time has been devoted to improving the experimental arrangement, and the work was carried far enough toward completion to seem to warrant a brief statement regarding the final arrangement.

The chief alteration from the set-up originally used was the enclosing of the specimen in a vacuum of such a quality as to appreciably reduce the loss of heat by convection. The accompanying figure shows the arrangement of parts. The only feature needing any comment is that used to make good thermal contact between the heating coil and specimen. This was satisfactorily accomplished by making the heating coil a helix through which the specimen would easily slip, and then sticking the two together with cæmentium. This substance proved itself a very firm bond, and up to 500° C. seems to be practically an electrical insulator. At somewhat higher temperatures it might be found to conduct to a troublesome extent.

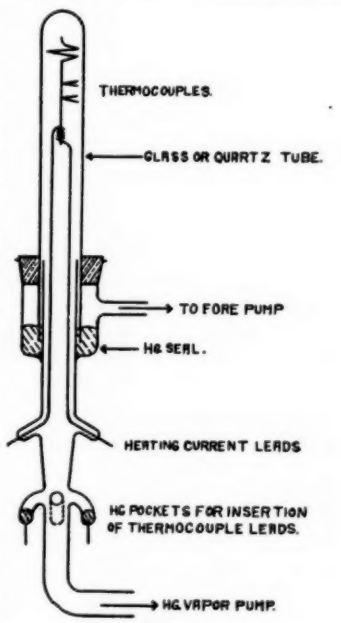

Fig. 1.

The pressure which the pump was able to maintain in the space around the specimen was never actually measured, but was sufficiently low to reduce the convection loss to not more than 1/10 of its value at atmospheric pressure.

Of course, evaporation sets rather definite upper limits to the temperatures at which various metals may be maintained *in vacuo*. The temperatures at which the evaporation would become so rapid as to interfere with the measurements of conductivity was determined in only a few cases. Tin and lead may be run to their melting points, while copper showed an appreciable but not serious evaporation at 400° C. Nickel can probably be run to 800° C.

The following table gives sample determinations on lead and copper, the only metals for which any final measurements *in vacuo* were made. The values of the specific heat given were taken from tables and are not to be

[1] Abstract of a paper presented at the Rochester meeting of the American Physical Society, October 26 and 27, 1917.

considered as applying accurately to the samples used. The copper specimen was a drawn wire 2.5 mm. in diameter, and the lead specimen was a "squirted" wire 3.1 mm. in diameter. The periods of the temperature variation used were 104 sec. and 235 sec.

TABLE.

Metal.	Distance Between Thermojunctions.	Temperature.	Specific Heat.	Conductivity.
Lead.....	2.42 cm.	90° C.	.0312	.0826
		92°	.0313	.0825
		92°	.0313	.0824
		210°	.0334	.0806
		210°	.0334	.0812
Copper.....	3.23 cm.	76°	.0937	.914
		84°	.0938	.917
		362°	.0997	.897
		362°	.0997	.882

WESTERN ELECTRIC CO.,
NEW YORK, N. Y.

ROTATION OF THE PULLEY IN MELDE'S EXPERIMENT.[1]

BY ARTHUR TABER JONES.

Introductory.—Let the string pass horizontally from the tuning fork to the pulley and then downward to the load, and let the prongs of the fork vibrate toward the pulley and away from it. Then if the fork vibrated slowly enough the load would move up and down and the pulley would rotate back and forth—both of them in phase with the prong to which the string was attached. With the frequencies actually used several observers[2] have noticed what looks like a continuous rotation of the pulley, but with the exception of a small amount of work[3] a few months ago no study of this rotation appears to have been made. For a particular fork, string, and pulley [fork making 100 double vib. per sec., string having linear density 2.24 mg./cm. and elastic modulus $50 \cdot 10^6$ dynes, pulley having radius 1.35 cm. and moment of inertia 19.8 g. cm.^{2}] this rotation is now studied.

The rotation may occur when the string is vibrating transversely or when it is not so vibrating, and the rotation may carry the top of the pulley toward the fork or away from it. Sometimes the rotation is as rapid as two or three turns in a second, but more often is much slower—frequently two or three turns in a minute. When the rotation carries the top of the pulley toward the fork call it a "rotation toward the fork." Let H and V mean respectively

[1] Abstract of a paper presented at the Rochester meeting of the American Physical Society, October 26 and 27, 1917.

[2] J. S. Stokes, PHYSICAL REVIEW, 30, p. 659, 1910. Raman and Apparao, PHYSICAL REVIEW, 32, p. 307, 1911. A. W. Porter, mentioned in Raman's paper.

[3] Jones and Phelps, PHYSICAL REVIEW, (2), 10, p. 541. 1917.

either the horizontal and the vertical parts of the string or the lengths of those parts.

Experimental.—With small loads and sufficiently great amplitudes of the fork it seemed to be always possible to obtain a very irregular motion of the string and simultaneously a very irregular rotation toward the fork.

When there was no transverse vibration of the string and when V was 20 cm. a sufficient amplitude of the fork caused rotation away from the fork for loads ranging from about 50 g. to 250 g. and for all lengths of H which were examined, viz., 10, 20, 30, 40, 50, 60, 70, 80, 100, 120, 150 cm. When V was 10 cm. or 30 cm. rotation away from the fork was observed when H had a number of these lengths—in two cases with loads running up to 700 g.—but when V was 40, 50, or 60 cm. this rotation was almost never observed. With no transverse vibration of the string a steady rotation toward the fork occurred for various lengths of V when H was 10, 20, or 30 cm. and the loads used were small—5 g. to 40 g.

When the string had a steady transverse vibration a series of observations in which $V = 10$ cm. and $H = 40$ cm. gave rotation away from the fork for the smaller amplitudes when there were three loops and what Melde called a "secondary tension," *i. e.*, a tension such that the transverse vibration of the string had a frequency which was the same as that of the fork. In every one of these cases and in no others a rotation away from the fork was observed. In each case an increase in amplitude changed the sense of the rotation. A similar series in which $V = 20$ cm. and $H = 40$ cm. gave similar results, except that the rotation away from the fork was observed in a very few other cases.

Theoretical.—The steady rotation of the pulley is probably in reality discontinuous—the string losing its hold on the pulley and allowing it to slip at a certain phase of the motion. This slipping would be most likely to occur when the tension of the string was small and the acceleration of the pulley large. If the minimum tension was reached when the top of the pulley was approaching one end of its path the successive slips would give rise to a net rotation toward that end of the path.

If we neglect the friction at the bearings of the pulley and assume that the string does not slip on the pulley and that there is no transverse vibration of the string, it is easy to set up the equation of motion for the system and to find a particular simple harmonic solution. Studies of the amount of the friction between the string and the pulley and between the pulley and its bearings made possible a rough correction of this solution. If the maximum difference which the solution indicates between the tensions in V and H is greater than the maximum difference which experiment has shown can exist without slipping, then at some phase of the motion there will be slipping and probably a net rotation. Now the solution shows that the ratio of the amplitudes of the tensions in V and H is independent of H—which checks the above experimental result for rotation away from the pulley. If the amplitude of the tension in V comes out in any case greater than that of the amplitude in

H the least tension in the string will occur when the tension in V is a minimum. If this is when the top of the pulley in its vibration is farthest from the fork any slipping that may occur will probably cause a net rotation away from the fork. Similar statements hold for other cases.

For ten cases in which rotation was observed the above calculations have been made. In all but one of them this reasoning indicates that a net rotation is to be expected, and for a majority of them the sense of rotation checks with that found experimentally.

A more complete theory must take account of the phase shifts produced by the friction at the bearings. The theory of the rotation when there is a transverse vibration of the string is not yet treated.

SMITH COLLEGE,
October 9, 1917.

COMPARATIVE ACCURACY OF WHIRLED PSYCHROMETER, ASSMANN ASPIRATION PSYCHROMETER, POROUS CUP ATMOMETERS, HAIR HYGROGRAPHS, PICHE EVAPORIMETER SATURATION DEFICIT RECORDER, OPEN WATER SURFACE EVAPORIMETER, AND DRY AND WET BULB THERMOMETERS.[1]

BY ALEXANDER MCADIE.

ATTENTION is called to the variety of names for instruments all essentially designed to indicate percentage of saturation of a mixture of atmospheric air and water vapor. It would be an advantage to group under one name all instruments used in studying evaporation and condensation in the free air. Note also that while the thermodynamics of the atmosphere has been more or less successfully studied, the hydrodynamics has been hurried over perhaps because of the absence of reliable data. In the various psychrometers (the word means a measure of the chilling due to evaporation) molecular energy change is shown as a fall in temperature. Various corrections are necessary and the method takes no account of nucleation. The sling psychrometer which is used as a standard of reference is subject to error in manipulation; and the writer is of the opinion that some other instrument should be used for this purpose. The whirled psychrometer as used officially is better, but unless a known volume of air and vapor mixture is used and some correction applied for the effect of centrifugal force, the readings are vitiated. An improvement has been made at Blue Hill by introducing a counter giving the number of revolutions per unit of time. This practically standardizes the velocity of air passing over the bulbs. All official humidity data thus far published need correction for personal error. The porous cup atmometer (Dr. Burton E. Livingston) is a simple device for measuring evaporation in cubic centimeters. It needs a recording device (one was added here) and gives records

[1] Abstract of a paper presented at the Rochester meeting of the Physical Society, October 26 and 27, 1917.

more consistent than the psychrometers. A black porous cup as well as a white have been used and as might be anticipated the evaporation is much larger whenever there is free absorption and radiation of heat. We are using in another atmometer wood alcohol in place of water to obtain a ratio which will serve in the winter months when water would freeze. The stationary dry and wet bulb is not satisfactory.

The data now extending over several months are designed for use in various ways but more particularly in an effort to evaluate a coefficient best representing the effect of the flow of air. While the temperature of the water, and of the air, percentage of saturation, atmospheric pressure and nucleation must be considered in any evaporation formula, at present the most promising line of investigation is the determination of the rapidity of removal of the water vapor; and this is effected chiefly by wind velocity and direction. Evidently pure diffusion effects are rare in nature and convection or mechanical removal is of chief importance. The wind effect is given by the last term in the equation

$$E = C(e_s - e_{d.p})(1 + \alpha V).$$

BLUE HILL OBSERVATORY OF HARVARD UNIVERSITY.

BOHR'S ATOM, ZEEMAN'S EFFECT AND THE MAGNETIC PROPERTIES OF THE ELEMENTS.[1]

BY JAKOB KUNZ.

ACCORDING to Bohr's theory the hydrogen atom consists of a nucleus with the elementary positive charge, surrounded by an electron in rotation. The electron is allowed to rotate in definite stationary orbits in which it does not radiate. Radiation occurs only, when the electron, moving from the outside toward the center, jumps from one stationary orbit to the next one. The strongest evidence in favor of this atom is the extraordinary coincidence of the calculated with the observed constant in the radiation formula of Rydberg. The laws of electrostatics are assumed to hold within the atom, while the laws of Maxwell's electromagnetic radiation are denied. The atom of hydrogen has a resultant moment of magnetism, and should therefore be paramagnetic. When 2 atoms combine to form a molecule, the resultant system must be as stable as possible. Among the three possible configurations the first seems to be more stable and is paramagnetic, the second, having no resultant moment, is diamagnetic, and the third of Bohr is paramagnetic. The experimental results seem to be contradictory; Quincke gave the positive value 0.008 at 1 atm. per c.c. while Bernstein found the diamagnetic value—0.005 and Bloudlot—0.034. This constant ought to be determined again. Helium in Bohr's theory possesses a remaining magnetic moment, giving rise to paramagnetism, but helium is decidedly diamagnetic.

[1] Abstract of a paper presented at the Rochester meeting of the American Physical Society, October 26 and 27, 1917.

Strange difficulties occur if we try to explain the Zeeman phenomenon on the basis of Bohr's hypothesis. H. A. Lorentz in his classical explanation of the simple Zeeman effect assumes a quasi-elastic force as a centripetal force

$$f \cdot r = 4\pi^2 n^2 r m.$$

and deduces by means of the ordinary magnetic action of a charge in motion the longitudinal and the transversal effect with the result:

$$\frac{e}{m} = \frac{(n_2 - n_1)2\pi c}{H},$$

a relation which has at first yielded an accurate value of e/m. In Bohr's theory, on the contrary, the centripetal force is equal to ee_1/r^2 for the non-radiating orbits. The fact that the magnetic field produces the Zeeman effect only in the presence of ionization, speaks in favor of Bohr's theory, and requires that the action only takes place when the electron is moving from one orbit to another one. For this transition a new assumption has to be introduced in order to account for the simple and for the more complicated effects. A certain compromise between Bohr's and Lorentz's theories seems to be necessary. So far the assumptions in both theories regarding the centripetal force are contradictory; yet Lorentz's theory gives the right value of e/m and the right kind of polarization, and Bohr's theory gives the right value of Rydberg's constant and very approximately the law of the series lines.

UNIVERSITY OF ILLINOIS,
URBANA, ILL.

THE INFLUENCE OF TEMPERATURE UPON THE CRUSHING STRENGTH OF A DENTAL AMALGAM. [1]

BY ARTHUR W. GRAY AND PARIS T. CARLISLE, 4TH.

WHILE dental amalgams are always used at the temperature of the human mouth, or approximately 37.5° C., strength tests of these important filling materials appear to have been made only at room temperatures. The authors have therefore determined the influence of temperatures between 25° and 95° C. upon the crushing strength of an amalgam prepared under carefully standardized conditions.

The need for such standardization was made evident by a series of preliminary experiments which showed the effect of variations in such factors as the proportions of mercury and alloy used in mixing the amalgam, the time devoted to triturating the mix, the temperature of trituration, the pressure under which the amalgam is molded into test cylinders, the time that this condensing pressure is maintained, the height of the test cylinder, the time that elapses between the making and the crushing of the cylinder, the temperature at which it is stored during this interval, and the rate at which the crushing load is applied during the testing.

[1] Abstract of a paper presented at the Rochester meeting of the American Physical Society, October 26 and 27, 1917.

In all the tests forming the subject of this communication the amalgam was prepared from a "balanced" alloy of the highest grade, that is to say, an alloy in which the proportions of the constituent metals are so adjusted that the expansion during hardening caused by the silver is almost, but not quite, neutralized by the contraction caused by the tin. This alloy contained approximately 68 per cent. silver, 26 per cent. tin, 5 per cent. copper, and 1 per cent. zinc. It was in the form of fine filings, just as furnished to the dentist. A weighed amount was incorporated with 1.60 times its mass of purified mercury by thoroughly triturating in a glass mortar for four minutes. The resulting smooth, plastic amalgam was quickly rolled into a ball and dropped into a thick-walled steel cylinder with a polished interior and an accurately fitting piston, upon which a load of 400 kg. was maintained for 8 minutes. This squeezed out the excess of mercury and condensed the amalgam, producing a cylinder 10.04 mm. in diameter by 11.5 mm. high, 40 per cent. of its mass being mercury. Cylinders prepared in this way were immediately placed in an incubator kept at 37.5° C., where they remained for several days before crushing, thus insuring completion of the hardening process.

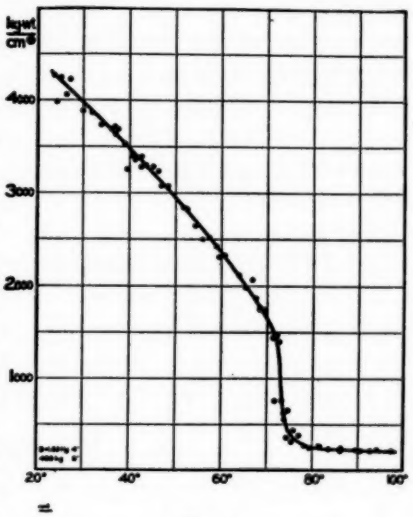

Fig. 1.

The crushing tests were made in a 9,000 kg. Olsen testing machine which was designed for operation by hand. The authors modified this by the addition of a motor drive which applied the load with regularity, thus permitting the beam to be kept balanced to a nicety right up to the moment of failure; and by electric heaters for bringing the specimen under test to the desired temperature, which was determined by a suitably placed thermoelement. An additional thermoelement enabled temperature gradients within the heated region surrounding the cylinder to be adjusted to negligible values.

The chart represents the individual results of crushing specimens at temperatures distributed fairly uniformly over the entire range. All the determinations made on three separate days are included. The abscissa of a point represents the temperature of a cylinder at the time it was being crushed; the ordinate the force in kilograms-weight sustained by the cylinder (which was 10.04 mm. in diameter) at the instant of failure. All of these points lie close to a curve which shows that with rising temperature the crushing strength of an amalgam prepared as described decreases somewhat faster than linearly from 5,300 kg. wt./cm.² at 25° to 4,050 at 45° and 2,550 at 65°. Soon after passing 70° the strength takes a sudden plunge and drops below 350 kg. wt./cm.² before 80° is reached. From this temperature up to 95° there is but little change

in strength, none of the cylinders having broken under stresses less than 260 kg. wt./cm.^{2}. At 37.5° the crushing strength was found to be 4,550 kg. wt./cm.^{2}, or nearly 65,000 lb. wt./in.^{2}.

The closeness with which all the points plotted (no observations in this three-day series have been omitted) follow a smooth curve shows the uniformity with which a dental amalgam can be prepared, and also the precision with which crushing tests can be made, provided proper precautions be taken.

The transition indicated by the rapid drop in strength between 70° and 75° has also been revealed by other methods. A more detailed account of the phenomena that occur in this temperature region will be presented in a future communication.

PHYSICAL RESEARCH LABORATORY,
THE L. D. CAULK COMPANY,
MILFORD, DELAWARE.

A NEW FORMULA FOR THE TEMPERATURE VARIATION OF THE SPECIFIC HEAT OF HYDROGEN.[1]

BY EDWIN C. KEMBLE.

NUMEROUS attempts to account for the rapid decrease in the specific heat of hydrogen at low temperatures on the basis of the quantum theory have thus far failed to yield a formula which is satisfactory from both the experimental and theoretical points of view. Several of the more promising theoretical formulas are open to objection in that they assume that the rotational specific heat of an assemblage of molecules each of which has two rotational degrees of freedom is twice that of a similar assemblage in which each molecule has but one rotational degree of freedom. The more recent work of

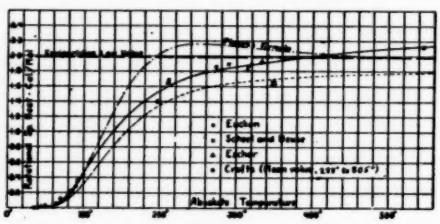

Fig. 1.

The rotational specific heat of hydrogen. The experimental points plotted are the observed values of C_v (reduced to the ideal gas condition in the case of the low temperature observations of Eucken, and Scheel and Heuse) minus 2.98, the value of C_v for monatomic gas.

Planck[2] on the application of the quantum theory directly to systems having more than one degree of freedom avoids this assumption, but the formula

[1] Abstract of a paper presented at the Rochester meeting of the American Physical Society, October 26 and 27, 1917.

[2] M. Planck, Verh. d. D. Phys. Ges., 17, pp. 407 and 438, 1915.

which he derives is not in agreement with the experimental facts. (See figure.)

Planck's discussion is based on his later form of the quantum theory in which the representative points of the various molecules in the state-space of statistical mechanics are assumed to be uniformly distributed throughout each individual region element. This form of the quantum theory can hardly be considered tenable at present in view of the observations of v. Bahr[1] (recently confirmed by Brinsmade and Kemble[2]) concerning the structure of the infra-red absorption bands of gases.

The writer has therefore adapted the methods used by Planck in the paper cited above to the older form of the quantum theory in which the representative points of the molecules are assumed to be confined to the bounding surfaces of the region elements. In order to bring the theory into complete harmony with the observed structure of the infra-red absorption bands of HCl and at the same time to derive a formula for the variation of the specific heat of a diatomic gas which would reproduce the observed values in the case of hydrogen, it was necessary to introduce the zero-point energy hypothesis in the form in which it occurs in the Bohr theory of atomic structure. This is done by excluding zero from the list of the possible values of the energy of rotation. It was also found to be necessary to take into account the increase in the moment of inertia of the molecules due to expansion under the influence of centrifugal force at the higher angular velocities. As a first approximation in correcting for the variation in the moment of inertia, it was assumed that the restoring force brought into play by a relative displacement of the nuclei is a linear function of the displacement.

The resulting formula for the rotational specific heat of a diatomic gas in calories per mol is

$$C_r = \frac{Nk}{J}\left[\frac{Q_3}{Q_1} - \left(\frac{Q_2}{Q_1}\right)^2\right].$$

Here

$$Q_1 = \sum_{n=1}^{\infty}(2n+1)e^{-\alpha_n/T},$$

$$Q_2 = \sum_{n=1}^{\infty}(2n+1)\alpha_n e^{-\alpha_n/T},$$

$$Q_3 = \sum_{n=1}^{\infty}(2n+1)\alpha_n^2 e^{-\alpha_n/T},$$

$$\alpha_n T = \frac{nh\nu}{2k}\, y_n(1 - y_n^2).$$

ν is the frequency of vibration of the atoms along their line of centers; y_n is the ratio of the frequency of rotation to the frequency of vibration, which

[1] E. v. Bahr, Phil. Mag., 28, p. 71, 1914.

[2] J. B. Brinsmade and E. C. Kemble, Proc. Nat. Acad. Sci., 3, pp. 420–425, June, 1917.

may be found by graphical solution of the equation

$$y_n \frac{(1 - y_n^2)}{(1 + y_n^2)^3} = \frac{nh}{4\pi^2 L_0 \nu}, \qquad n = 1, 2, 3, \cdots.$$

L_0 is the moment of inertia for zero angular velocity and J is the mechanical equivalent of heat. The other symbols have their usual meanings.

Giving L_0 the value 2.0×10^{-41} gm. cm.^{2} and ν the value 1.2×10^{14} sec.$^{-1}$, we obtain the curve A of the accompanying figure. The curve B shows the theoretical specific heat for rigid molecules ($\nu = \infty$) with the same value of L_0.

It is perhaps desirable to emphasize the fact that, though the above formula contains the frequency of vibration, it gives the rotational specific heat only. At temperatures above 500° or 600° the vibrational specific heat also must be taken into account.

BUFFALO, NEW YORK.

NEW BOOKS.

Practical Pyrometry. The Theory, Calibration and Use of Instruments for the Measurement of High Temperatures. By Ervin S. Ferry, Glenn A. Shook, and Jacob R. Collins. New York City, John Wiley & Sons. Pp. vii + 143, $5\frac{1}{2} \times 8$. Cloth, $1.50.

The present book was written for three classes of readers—"college students, technically trained men who deal with processes requiring high temperature measurements, and less trained observers who may make the measurements." The book is practically a synopsis of "Measurement of high temperatures" by Burgess and Le Chatelier, Wiley, 1912, but is presented in different form especially suitable for the class room, as minor details are omitted, and in several places the text is illustrated by practical problems. Also at the end of each chapter are several experiments, fourteen in all, which are prepared in sufficient detail for the ordinary student, and which cover the field of pyrometry very satisfactorily. The chapters are headed as follows: (1) Standard Temperature Scales; (2) Resistance Pyrometry; (3) Thermoelectric Pyrometry; (4) Radiation Pyrometry; (5) Optical Pyrometry. The text is well illustrated and several new American instruments are shown. A few comments may be made on points of minor detail. In the preface it is stated that "the day is already past when foundrymen and steel workers depend upon the eye to judge the temperatures of their product in the various stages of its heat treatment, when makers of ceramic products depend upon the indication of fusible cones," etc. One needs but visit industrial plants to realize this utopian condition is far from being fulfilled. Probably nine out of ten ceramic industries employ fusible cones or similar means of temperature measurement, and many of the leading ceramic engineers of this country advocate their use in preference to more scientific instruments. A point in history is brought out on page 3. Bolton ("Evolution of the thermometer") states that Celsius assigned the number 100 to the temperature of melting ice and 0 to the temperature of steam. The present assignment of numerals was made by Christ in 1743. Also, according to Bolton, Fahrenheit did not assign the number 212 to the boiling point of water as here stated. The method of correcting for lead resistance of the resistance thermometer, page 21, is crude. Even for the most elementary students, the bridge should be arranged as in Fig. 7. A student will be interested in solving the mathematics of the Wheatstone bridge in order to see why the arrangement in Fig. 7 compensates properly. Sulphur should not be boiled in an aluminum tube as illustrated in Fig. 15. With such a tube electrically heated to the top as shown in the figure, the vapor can be superheated to almost any value. The heating coil should be

much shorter, and for accurate work glass tubes are to be preferred. The geometrical optics of Fig. 44 is incorrect. Prism *M*, Fig. 62, should be turned through 180°. Recent work indicates that C_2 is more nearly equal to 14,350 than 14,500 as given on page 91. On page 140 it is stated that "a person of no training can get better results with a radiation pyrometer than with an optical pyrometer." This is contrary to experience. Published investigation has shown that persons who are totally unfamiliar with the optical pyrometer can set to within 5° or 10° C. To obtain such accuracy with a radiation pyrometer requires a great amount of experience and a consideration of many factors which are not mentioned in any text-book. Table 3, page 143, giving $\log \tan^2 \vartheta$ is unnecessary. In plotting data for the Wanner pyrometer, it is more convenient to use $\log \tan \vartheta$ versus $1/T$. The demand for engineers having some knowledge of practical pyrometry is becoming greater every year. Many schools are offering courses in this subject and the day is near when pyrometry will be a required course for engineers and chemists. The present book should serve as a suitable text for a junior or senior course covering one semester.

P. D. F.

Recreations in Mathematics. By H. E. LICKS. New York: D. Van Nostrand Co., 1917. Pp. v + 155. Price, $1.25.

This book is to entertain and to arouse in students and amateurs an interest in mathematics. In the few pages devoted to mechanics and physics, the teaching of mechanics by the physicist is criticized and the following statement made in apparent seriousness: "Surely the subjects of heat, light, sound and electricity furnish a sufficient field for the physicist, without encroaching on the topic of mechanics, which properly belongs to the engineer."

F. B.

The Principles of Electric Wave Telegraphy and Telephony. By J. A. FLEMING. New York: Longmans, Green and Co., 1916. Pp. xvi + 911. Price, $10.00.

The original book of 671 pages (first edition, 1906), expanded in a second edition (1910), is now further extended, the total expansion of 240 pages being due in part to the re-writing of portions of the work and to additions in the several chapters, and in part to added chapters on Transmission of Radiotelegraphic Waves over the Earth and on Radiotelephony. As in the earlier editions, it has been the aim of the author to deal chiefly with principles and not to devote much space to details of apparatus. Between the covers of the book is an immense amount of valuable material.

F B.